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NON-DIMENSIONAL PARAMETRIC ANALYSIS FOR THE SEISMIC RESPONSE OF DUAL MOMENT-RESISTING AND BUCKLING-RESTRAINED BRACED FRAMES

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Abstract: *Buckling-restrained braces (BRBs) have proven to be very effective devices improving the seismic performance of existing and new building frames. They provide strength, stiffness and added damping to the structure, however, due to their low lateral post-elastic stiffness, their use may lead to excessive residual deformations which may hinder the building's reparability. Moreover, excessive cumulative ductility demand in the BRBs may compromise the capability of withstanding multiple earthquakes. To overcome these drawbacks, BRB frames (BRBFs) can be coupled with moment-resisting frames (MRFs) to form a dual system. If properly designed, the MRF acts as a back-up frame and allows to control the residual drifts and optimize the performance of the BRBs. This paper attempts to provide insights into the performance and residual capacity of this type of dual systems and to shed light on the influence of the main BRB's design parameters. A non-dimensional formulation of the equation of motion is derived and an extensive parametric study is carried out on a single-degree-of-freedom system subjected to a set of natural records with different characteristics and scaled to various intensity levels. This allows to investigate a wide range of configurations, considering different levels of the relative strength of the BRBF and MRF and their ductility demand, and to obtain useful information for the BRBs design.*

Introduction

Buckling-restrained braces (BRBs) are elastoplastic energy dissipation devices (e.g., Soong and Spencer 2002) and can be employed to resist the seismic-induced forces and dissipate the seismic energy in new constructions and for the rehabilitation of existing buildings. In BRBs, a sleeve provides buckling resistance to an unbonded core that resists the axial stress. As buckling is prevented, the core of the BRB can develop axial yielding in both tension and compression with an almost symmetric hysteretic behaviour.

BRBs provide large and stable dissipation capacity (e.g., Black et al. 2002), however, they are characterised by low post-yield stiffness which may result in inter-story drift concentration (e.g., Zona et al. 2012) and large residual inter-storey drifts. The latter problem is associated with high repair costs and disruption of the building use or occupation (e.g., Erochko et al. 2010). Sabelli et al. (2003) studied the seismic performance of buckling-restrained braced frames (BRBFs) reporting residual drifts on average in the range of 40 to 60% the peak drift. These values, of the order to 1.6 to 2.4 % for a peak drift of 4%, could be quite high considering that building reparability may be compromised for residual drifts higher than 0.5% (e.g., McCormick et al. 2008). In addition, high residual drifts due to a mainshock could jeopardize the frame performance under aftershocks.

The above issue, which may impair the cost-effectiveness of BRBFs, could be controlled by placing a steel moment-resisting frame (MRF) in parallel with the BRBF to create a dual system as schematically represented in Figure 1(a) (e.g., Kiggins and Uang 2006, Ariyaratana and Fahnestock 2011, Baiguera et al. 2016). According to ASCE/SEI 7-10 (2010), in dual systems, the MRF should be capable of resisting at least 25% of the prescribed seismic force. Kiggins and Uang 2006 investigated the seismic response of a 3- and a 6-storey BRBFs with and without parallel MRFs designed to resist 25% of the design base shear, showing a reduction of the residual drifts by about 50%. Similarly, Ariyaratana and Fahnestock (2011) investigated a 7-storey

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dual BRBF-MRF system case study finding similar result. Beside this, BRBs can be also employed for retrofitting existing reinforced concrete MRFs (e.g., Freddi et al. 2013), forming a dual systems.

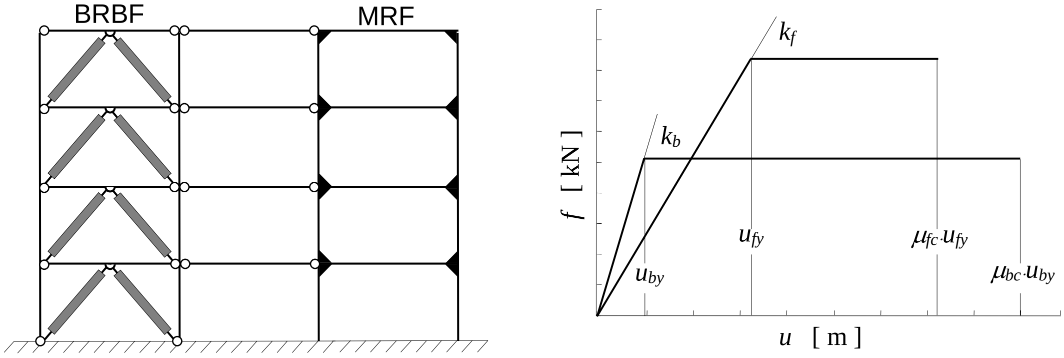

Figure 1. (a) Schematic dual system combining a buckling-restrained braced frame (BRBF) and a moment resisting frame (MRF); (b) Constitutive law of the SDOF dual systems.

The works reported above evaluated the efficiency of dual BRBF-MRF systems by considering only few case studies, without providing general information about the influence on the seismic performance of the shear ratio, stiffness ratio and target design ductility of the two systems. In this work, an extensive parametric investigation is carried out to shed light on the influence of these parameters and to provide useful recommendations for the preliminary design. The problem is analysed by assuming that both the BRBF and the MRF can be described as single degree of freedom (SDOF) systems. This representation, which is suitable under some regularity conditions (e.g., Ragni et al. 2011, Maley et al. 2010), allows to derive a non-dimensional formulation of the problem and highlight the characteristic parameters that control the seismic performance. The analysis is performed by varying the non-dimensional parameters and allow to explore the performance of a wide range of configurations under a set of ground motion records accounting for seismic input uncertainty.

Engineering demand parameters (EDPs) of interest include the normalized peak responses, the normalized residual displacements, and the BRBs cumulative ductility demand. These EDPs are evaluated in correspondence of the design condition, where the BRBF and MRF attain simultaneously their target ductility capacity. The results of the parametric study provide the median values, estimated by the geometric mean (GM), of the monitored EDPs. The variability of the EDPs, controlled by lognormal standard deviation, is not considered due to space constraints. Different design choices, corresponding to various combinations of the BRBF and MRF ductility demand are investigated, in order to obtain some results that can be useful for the design of the BRBF.

Problem formulation

Equation of motion

The equation of motion governing the seismic response of a SDOF system representative of a dual system can be expressed as:

$$m\ddot{u}(t) + c_f\dot{u}(t) + f_f(t) + f_b(t) = \ddot{u}_g(t) \tag{1}$$

where m and c_f denote respectively the mass and the viscous damping constant of the system, $f_f(t)$ the resisting force of the MRF, $f_b(t)$ the resisting force of the BRBF while $\ddot{u}_g(t)$ represents the ground acceleration input.

The MRF is assumed to have an elastoplastic behaviour, with initial stiffness k_f, yield displacement u_{fy} and ductility capacity μ_{fc}. Differently, the BRBF has a constitutive law described by the constitutive model developed by Zona and Dall'Asta (2012). This model is characterized by several parameters that allow to accurately describe the devices' hardening and hysteretic behaviour. In this work, only the variability of the BRBs initial stiffness k_b, yield displacement u_{by} and ductility capacity μ_{bc} is considered. These parameters are the ones which exhibit significant variation from device to device and that are explicitly reported in catalogues. The two models working in parallel, whose constitutive behaviour is illustrated in Figure 1(b), are representative of the dual SDOF system. Such a model can describe a wide range of structural configurations of dual BRBF-MRF systems typical of both new (Kiggins and Uang 2006, Ariyaratana and Fahnestock 2011, Baiguera et al. 2016) and retrofitted frames (e.g.*,* Freddi et al. 2013).

The seismic input is characterized by significant uncertainty affecting not only its intensity, but also the duration and frequency content. As usual in Performance Based Earthquake Engineering, the uncertainty of the seismic input is treated by introducing a seismic intensity measure (*IM*) (Freddi et al. 2017) whose statistical description is the object of the hazard analysis. The ground motion randomness for a fixed intensity level *im*, can be described by selecting a set of ground motion realizations characterized by a different duration and frequency content and scaling these records to the common *im* value. The system response for a ground motion with an intensity *im* can be expressed as:

$$m\ddot{u}(t)+c_f\dot{u}(t)+f_f+f_b=im\cdot\bar{\ddot{u}}_g(t) \tag{2}$$

where $\bar{\ddot{u}}_g(t)$ denotes the ground motion records scaled such that im = 1 for that record.

The choice of an appropriate *IM* for the problem should be driven by criteria of efficiency, sufficiency, and hazard computability (e.g.*,* Tubaldi et al. 2015, Freddi et al. 2017). In this paper, the spectral acceleration $S_a(\omega_0, \xi)$, at the fundamental circular frequency of the system, ω_0, and for the damping factor ξ is employed as *IM.*

Non-dimensional formulation

Based on Equation 2, the maximum relative displacement of the system, $u_{\max}$, under the fixed ground motion with history $\bar{\ddot{u}}_g(t)$, can be expressed as:

$$u_{\max}=f\left(m,c_f,k_f,u_{fy},k_b,u_{by},im\right) \tag{3}$$

The 8 variables appearing in Equation 3 have dimensions: [$u_{\max}$] = L, [m] = M, [c_f] = MT^{-1}, [k_f] = ML^{-2}, [u_{fy}] = L, [k_b] = ML^{-2}, [u_{by}] = L, [im] = LT^{-2} where the 3 physical dimensions are the time T, the mass M, and the length L. By applying the Buckingam Π-theorem (Barenblatt 1987), Equation 3 can be conveniently reformulated in terms of dimensionless parameters, denoted as Π-terms identifying the parameters that control the seismic response of the system. The problem involves 3 physical dimensions and 8 dimensional variables, thus, only 8 - 3 = 5 Π dimensionless parameters are needed. By selecting the m, im and k_f as repeating variables, the Π-terms can be derived and, after manipulation, the following alternative set of Π-terms can be obtained:

$$\delta=\frac{u_{\max}\omega_0^2}{im}\ ,\ \mu_f=\frac{u_{\max}}{u_{fy}}\ ,\ \mu_b=\frac{u_{\max}}{u_{by}}\ ,\ \xi=\frac{c_f}{2m\omega_0}\ ,\ \alpha=\frac{f_b}{f_f} \tag{4}$$

where $\omega_0^2 = (k_b + k_f)/m$ denotes the square of the circular frequency of the SDOF dual system. The parameter δ denotes the displacement demand $u_{\max}$ normalized with respect to the displacement value im/ω_0^2. By considering $S_a(\omega_0, \xi)$ as *IM*, it can be interpreted to as the displacement amplification factor, being the ratio between $u_{\max}$ and the pseudo-spectral displacement $S_a(\omega_0, \xi) = S_a(\omega_0, \xi)/\ \omega_0^2$. The parameters μ_f and μ_b denote the ductility demand respectively of the MRF and the BRBF. The parameter α, represents the ratio between the strength capacity of the bracing system and of the frame (*e.g.,* Freddi *et al.* 2013). According to the ASCE/SEI 7-10 (2010), in a dual system, α should be lower than α_d = 3.

It is important to observe that, while the parameters μ_f, μ_b and δ of Equation 4 depend on the response of the system through $u_{\max}$, the parameters α and ξ are independent from the response. Other EDPs of interest for the performance assessment can be derived from the non-dimensional solution. In particular, the following EDPs are considered:

$$\mu_{b,cum}=\frac{u_{bp,cum}}{u_{by}}\ ,\ \delta_{res,el}=\frac{u_{res}\omega_0^2}{im}\ ,\ \delta_{res}=\frac{u_{res}}{u_{\max}}\ ,\ \alpha_{abs}=\frac{a_{\max}}{im} \tag{5}$$

where $\mu_{b,cum}$ denotes the normalized cumulative plasticity (*i.e.*, ductility) demand of the BRBF, $\delta_{res,el}$ the ratio between the residual displacement and the peak spectral displacement of the system, δ_{res} the ratio between the residual and the peak inelastic displacement of the system, and α_{abs} the absolute acceleration a_{max} normalized by the seismic intensity im.

The system response in terms of these EDPs depends on the characteristics of the input via the circular frequency ω_0. In fact, seismic inputs with the same intensity *im* but with different characteristics propagate differently and have different effects on systems with different natural frequencies ω_0 (see e.g. Tubaldi et al. 2015).

Performance assessment methodology

The objective of the proposed study is to evaluate how the coupled system behaves in correspondence of the design condition, defined by the target ductility levels of the BRBF and the MRF (e.g., Freddi et al. 2013, Zona et al. 2012), respectively μ_{bt} and μ_{ft}, corresponding to the design earthquake input. More specifically, assuming a target ductility level $\mu_{bt} \leq \mu_{bc}$ for the BRBF and a target ductility level $\mu_{ft} \leq \mu_{fc}$ for the MRF, the design condition corresponds to $\mu_b = \mu_{bt}$ and $\mu_f = \mu_{ft}$ under the assumed seismic input. For example, the BRBF may be designed to attain a ductility demand μ_{bt} = 10, while the MRF may be designed to remain elastic with μ_{ft} = 1.

The above design criterion imposes a constraint on the values that can be assumed by the non-dimensional problem parameters, which makes the methodology different from those followed in other researches on similar systems employing non-dimensionalization and considering free parameters variations (Karavasilis et al. 2011, Tubaldi et al. 2015, Málaga-Chuquitaype 2015). Given the system properties independent from the response ω_0, α, μ_{ft}, μ_{bt}, ξ, the design condition can be found by the following optimization problem: find the value δ^* of the normalized displacement demand such as $\bar{\mu}_f = \mu_{fc}$ and $\bar{\mu}_b = \mu_{bc}$, where the over score denotes the mean across the samples, and thus $\bar{\mu}$ denotes the mean ductility demand. The following procedure can be applied to ensure the attainment of the design condition under the set of records employed to describe the seismic input:

1. Assign arbitrary values to the target mean displacement demand $\bar{u}^*_{\max}$ and to *m*, e.g., $\bar{u}^*_{\max}$ = 1 m and *m* = 1 ton. The corresponding non-dimensional parameter values are: $c_f = 2m\omega_0\xi$, $u_{fy} = \bar{u}^*_{\max} / \bar{\mu}_f$, $u_{by} = \bar{u}^*_{\max} / \bar{\mu}_b$, $k_f = \omega_0^2 m / \left(1 + \alpha u_{fy} / u_{by}\right)$, $k_b = \left(\alpha u_{fy} / u_{by}\right) k_f$;
2. Scale the records to a common value of the intensity measure e.g., *im* = 1;
3. Perform nonlinear dynamic analyses for the different records;
4. Evaluate the mean system displacement response $\bar{u}_{\max}$. If $\bar{u}_{\max}$ is equal to the target value $\bar{u}^*_{\max}$, then $\delta = \delta^*$, where $\delta^* = \bar{u}^*_{\max}\omega_0^2 / im$, and go to step 5. Otherwise multiply *im* by the ratio $\bar{u}^*_{\max} / \bar{u}_{\max}$ and restart step 2. This procedure corresponds to a linear interpolation between the relation $\bar{u}_{\max}$ and *im*. If this procedure does not converge, resort can be made to any optimization algorithm;
5. Evaluate the statistics of δ_{res}, $\delta_{res,el}$, α_{abs}, and $\mu_{b,cum}$.

Steps 1-4 ensure that the design condition of the MRF and the BRBFs attaining simultaneously their performance target under the design earthquake input is achieved.

Parametric study

System properties

The parametric analysis is performed for different values of ω_0, α, μ_{ft}, μ_{bt} considering the constraint posed by the attainment of the design condition (*i.e.,* $\delta = \delta^*$). The parameter ω_0 is varied in a range corresponding to a vibration period $T_0 = 2\pi/\omega_0$ between 0 and 4 sec. The strength ratio α assumes the values in the range between 0 and 100 where the lower bound α = 0 represents the case of the bare frame, whereas the upper bound represents of BRBF only. The parameter μ_{ft} assumes values between 1 and 4 where for μ_{ft} = 1 the frame behaves elastically under the design earthquake while for μ_{ft} = 4 a highly ductile behaviour of the frame is expected. The parameter μ_{bt} assumes values in the range between 5 and 20. Values of 15-20 are typical ones of a BRB device, however, when the BRB device is arranged in series with an elastic brace (i.e., Freddi et al. 2013) the ductility capacity may attain the lower bound of 5. The value of 2% is assumed for the damping factor ξ in this study.

Seismic input description

A set of 28 ground motions is considered in the parametric study to describe the record-to-record variability. The records have been selected from the PEER strong motion database (FEMA P695) considering site class B, as defined in Eurocode 8, source-to-site distance, R, greater than 10 km, and a moment magnitude, M_w, in the between 6.0 and 7.5.

Parametric study results

Figure 2 shows the GM of the normalized peak displacement demand δ vs the base shear ratio α, for different values of the target BRBF ductility μ_{bt}. The different figures refer to different values of T_0 and of the target frame ductility μ_{ft}. All the curves attain the same value for $\alpha = 0$ (MRF only), and in particular for $\mu_{ft} = 1$ they attain a value of about 1. This result is expected, since for $\alpha = 0$ the response is not dependent on the BRBF's ductility capacity, and for $\mu_{ft} = 1$ the system behaves (on average) elastically, so that the inelastic displacement coincides with the elastic one. On the other hand, for $\alpha = 0$ and $\mu_{ft} = 4$, a simple bilinear oscillator is obtained and δ can be significantly different than 1. In particular, higher values of the normalized peak displacement δ are observed for low values of the period T_0. In the case of dual system ($\alpha > 0$), for low periods and increasing values of α, the normalized peak displacement increases, whereas for high periods δ remains almost constant and slightly less than 1.

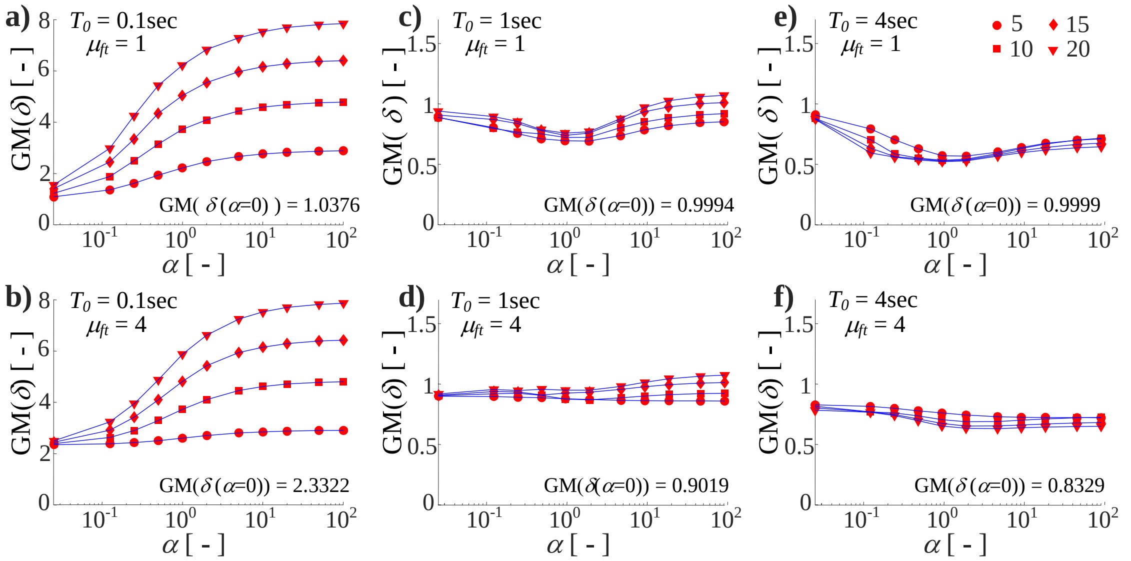

Figure 2. Geometric mean of the normalized peak displacement demand δ vs the base shear ratio α, for different values of T_0 (0.1, 1 and 4 sec), of μ_{ft} (1 and 4) and of μ_{bt} (5, 10, 15 and 20).

Figure 3 shows the GM of the normalized residual displacement demand δ_{res} vs the base shear ratio α, for different values of the target BRBF ductility μ_{bt}. The different figures refer to different values of T_0 and of the target frame ductility μ_{ft}. In general, the period T_0 does not influence significantly this response parameter. Moreover, as expected, when the system behaves linearly ($\alpha = 0$, $\mu_{ft} = 1$), the residual displacement is zero. Obviously, adding in parallel to a linear system a nonlinear one ($\alpha > 0$ in Figure 3(a), (c) and (e)) results in an increase of residual displacements. This increase is higher for higher values of the target BRBF ductility μ_{bt}. Thus, high values of α must be avoided to limit residual displacements. The limit $\alpha_d = 3$ posed by SEI/ASCE 7-10 (2010) on the maximum values of α in dual systems provides a good control of the residual displacements, ensuring values of δ_{res} lower than 0.25. On the other hand, if the frame exhibits a nonlinear behaviour with a target ductility $\mu_{ft} = 4$, then it is characterized by high residual drifts of the order of 40% of the peak ones, and adding in parallel the BRBs ($\alpha > 0$ in Figure 3(b), (d) and (f)) does not increase them significantly.

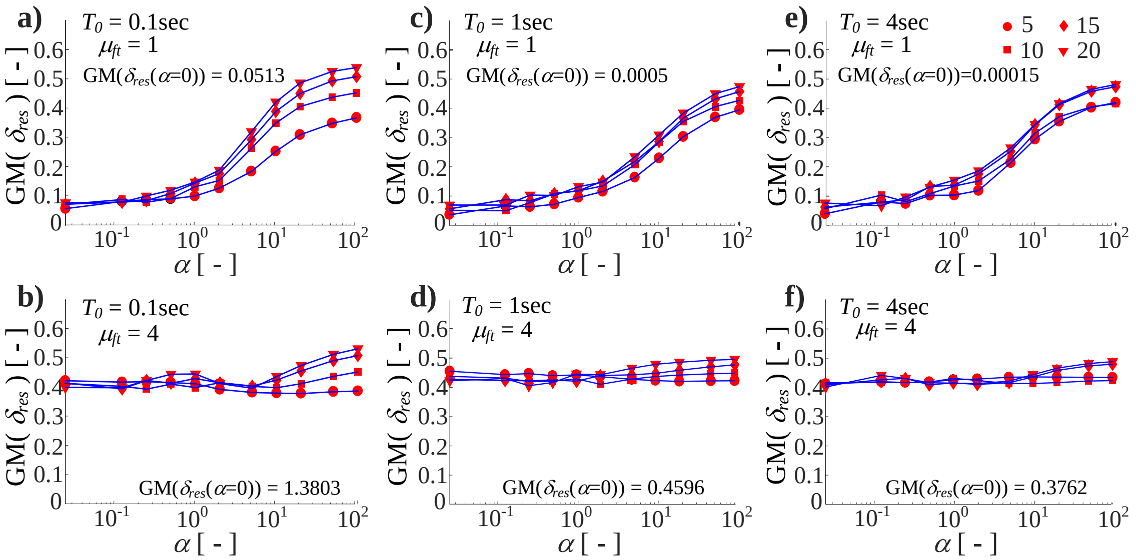

Figure 3. Geometric mean of the normalized residual displacement demand δ_{res} vs the base shear ratio α, for different values of T_0 (0.1, 1 and 4 sec), of μ_{ft} (1 and 4) and of μ_{bt} (5, 10, 15 and 20).

Figure 4 shows the GM of the cumulative plastic ductility demand in the BRBF $\mu_{b,cum}$ vs the base shear ratio α, for different values of the target BRBF ductility μ_{bt}. The different figures refer to different values of T_0 and of the target frame ductility μ_{ft}. In general, the cumulative ductility demand reduces by increasing α. This can be explained by observing that the system becomes more nonlinear by increasing α, and this generally results in fewer cycles at the maximum deformation and less ductility accumulation under an earthquake history, as it can be seen in Figure 4 for a specific case and earthquake record. The cumulative ductility increases with the target ductility level and this increase is different for the different period considered. Moreover, the results show an almost linear relation between $\mu_{b,cum}$ and μ_{bt}, hence the curves for $\mu_{b,cum}/\mu_{bt}$ collapse into a single master-curve. These trends suggest that higher α values permit to control better the cumulative ductility demand in the BRBs. However, the decrease is higher for low α values, whereas for $\alpha > \alpha_d$ the cumulative ductility does not decrease significantly.

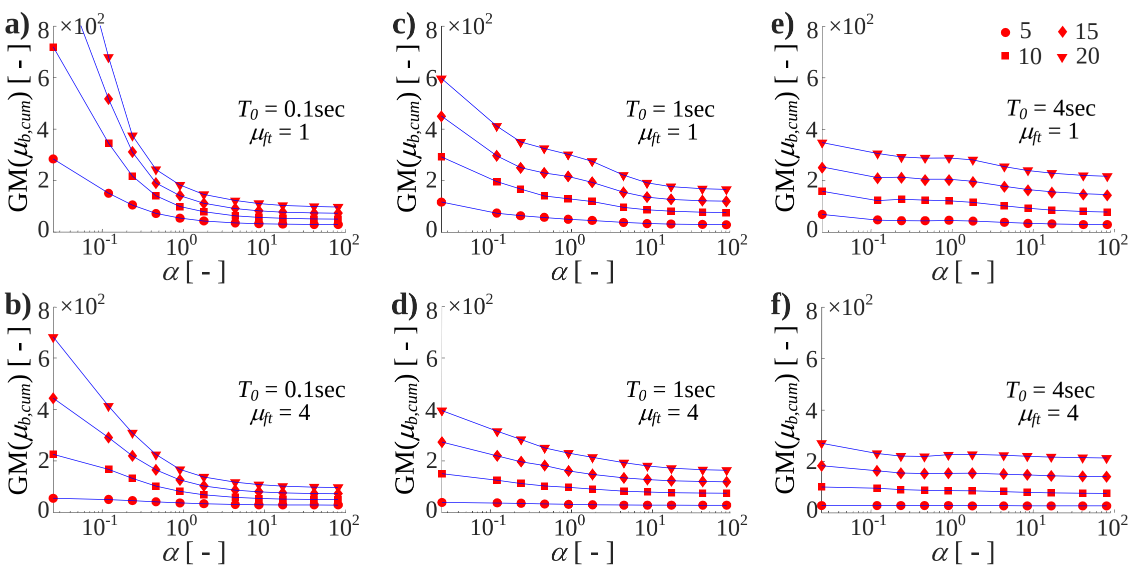

Figure 4. Geometric mean of the BRBF's normalized cumulative ductility demand $\mu_{b,cum}$ vs the base shear ratio α, for different values of T_0 (0.1, 1 and 4 sec), of μ_{ft} (1 and 4) and of μ_{bt} (5, 10, 15 and 20).

For the sake of brevity, the trends of variation of the normalized absolute acceleration α_{abs} are not shown here. The GM values of α_{abs} are equal to one for the linear system, and reduce by increasing the level on nonlinearity of the system, i.e., by increasing the values of α, of μ_{bt}, and of μ_{ft}. For very short period systems, α_{abs} increases as the period decreases because the peak absolute acceleration eventually approaches the peak ground acceleration as T tends to zero (Karavasilis and Seo 2011). It is noteworthy that for values of α increasing beyond 1, the absolute acceleration does not decrease significantly.

Conclusions

This paper presented the results of a study on the seismic performance of dual systems consisting of BRBFs coupled with MRFs, designed according to a criterion which aims to control the maximum ductility demand on both the resisting systems. A single degree of freedom system assumption and a non-dimensional problem formulation allow estimating the response of a wide range of configurations while limiting the number of simulations. This permits to evaluate how the system properties affect the median demand and the dispersion of the normalized displacements, residual displacements, normalized accelerations and cumulative BRB ductility.

The results of this study show that adding a very ductile BRBF in parallel to a MRF may result in excessively residual displacements, particularly for high values of the BRBF to MRF strength ratio α. The limit α_d = 3 posed by SEI/ASCE 7-10 on the maximum values of α in dual systems yields values of the median residual-to-peak displacements of the order of 0.15-0.20, which may be excessive in some situations. In fact, considering a maximum inter-storey drift ratio of 4%, the expected residual drift would be of the order of 0.60%-0.80%, which is higher than the limit of reparability. Moreover, it can be observed that the median cumulative ductility demand of the BRBFs has an opposite trend of variation with α compared to the residual displacement, i.e., it decreases by increasing α. This result has also an impact on the choice of α in the design, since excessive accumulation of plastic deformations affect the safety of the BRBs and their capability to withstand aftershocks and multiple earthquakes.

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