COMPARISON OF NATURAL AND SYNTHETIC SPECTRUM COMPATIBLE ACCELEROGRAMS OBTAINED BY GROUND MOTION SELECTION AND STOCHASTIC SIMULATION

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Abstract: The accurate modelling of seismic load is a major topic that has raised particular interest in the literature in the recent years. One of the reasons is the advent of performance based earthquake engineering (PBEE) that has become the state of the art for both civil and nuclear structures. The PBEE generally requires transient analysis in order to evaluate the seismic fragility of structures and components. In consequence, a set of hazard consistent ground motion time histories is needed for the transient dynamic analysis to be performed.
In this paper, a simple procedure for simulating artificial earthquake accelerograms matching the statistical distribution of response spectra, as given by the ground motion prediction equations (median and the standard deviation) and correlation coefficients, is presented. The approach, Zentner (2014), follows the general ideas of the natural ground motion selection algorithms proposed by Baker (2011) and Wang (2011) but using simulated (artificial) "spectrum-compatible" accelerograms. This contribution proposes to perform a number of comparative studies in order to assess the capabilities of the simulated accelerograms. Ground motion intensity measures will be compared to the target from GMPE and to the ones of selected natural accelerograms. Finally, the impact on structural response will be evaluated. In particular, we compare a set of natural accelerograms, selected according to the Baker et al. procedure, to a set of simulated time histories.

Introduction

The selection of pertinent input ground motions for transient structural analysis is a difficult task that has to be addressed thoroughly in the framework of performance-based earthquake engineering. It is now widely acknowledged that not only the mean or median spectrum, but also the accurate spectral shape is an important issue.
Several methodologies for obtaining spectrum-compatible ground motion time histories, to be used in transient dynamic structural analysis, are available in literature. In particular, Abrahamson et al (2010) proposed to modify natural ground motion by time domain wavelets so as to perfectly match a target spectrum. This methodology is available through RSPMatch software. Another widely used methodology, based on the so-called Random Vibration Theory (RVT), is due to Vanmarcke. It is a stochastic model where a spectrum-compatible PSD is identified by using the relation between the standard deviation and the distribution of the maxima of a stationary Gaussian stochastic process. In order to improve the matching, the spectral content of the sample of time histories is subsequently adjusted in the frequency domain. This approach has been distributed via SMQKE software, Gasparini&Vanmarcke 1976. POWERSPEC, a software based on the work of Preumont 1985 used in the nuclear industry, as well as *Code_Aster*, propose a similar approach for the simulation of spectrum compatible ground motion. Recently, Baker and co-workers (2011) have proposed a new methodology for ground motion selection and possibly scaling that allows to obtain sets of spectrum compatible ground motion exhibiting realistic spectral shape. This approach is of particular interest when the target is a conditional mean spectrum, Baker (2011). Indeed, the notion of conditional mean spectrum (CMS) has been introduced in the framework of performance based safety assessment methods in order to provide a spectral shape compatible with specific event scenarios obtained by disaggregation from the Uniform

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Hazard Spectra (UHS). It provides a suitable spectral shape and can be used for record selection or generation, Baker (2011).
The rationale of the approach proposed in Zentner (2014) is to combine the record selection procedure of Baker with spectrum-compatible PSD models in order to simulate accelerograms matching various spectral shapes, in particular conditional mean spectra. However, being artificial accelerograms, obtained from a stochastic model, the question of the representativeness with respect to natural events arises. This topic is analyzed and discussed in this paper. In particular, we compare a set of natural accelerograms, selected according to the Baker et al. procedure, to a set of simulated time histories. Ground motion intensity measures will be compared to the target from GMPE and to the ones of selected natural accelerograms.
This paper is organized in two parts. First of all, the ground motion simulation procedure is presented with more detail. Secondly, sets of artificial accelerograms, ontained by the stochastic ground motion simulation model, are compared to sets of natural accelerograms selected by the procedure and algorithms proposed by Baker and coworkers.

Ground motion simulation methodology

According to the Ground Motion Prediction Equations (GMPE), the response spectra are lognormal stochastic process defined by its median, log standard deviation and correlation coefficients. The ground motion selection algorithm of Baker (2011), further developed in Jayaram et al. (2011), is based on the simulation of realizations of the target response spectrum and the selection of natural accelerograms that best match these spectra. The matching is verified one-by-one, that means that for each simulated response spectrum the best matching natural accelerogram is chosen. The software (Matlab programs) for this methodology is provided by the authors and available for download on http://web.stanford.edu/~bakerjw/gm_selection.html.
Clearly, this approach allows to adequately match the spectral shape given by the GMPE. Moreover, this procedure can be also used for the simulation of scenario specific artificial time histories. This is described with more detail in what follows.

The target log-spectral acceleration $\ln S_a(T)$ at periods $T_1, T_2, \cdots T_n$ can be modeled by a Gaussian random vector $\mathbf{Z} \sim \aleph(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where the mean value $\boldsymbol{\mu}$ is given by the median attenuation model and the term Σ_{ij} of the covariance matrix $\boldsymbol{\Sigma}$ can be evaluated using the correlations coefficients, Baker& Jayaram (2008):

$$\Sigma_{ij} = \rho\left(T_i, T_j\right)\sigma_{\ln Sa\,(T_i)}\sigma_{\ln Sa\,(T_j)}, \quad i, j = 1, 2 \cdots n. \tag{1}$$

The numerical simulation of realization of the correlated Gaussian random vector can be achieved by using the classical formula:

$$\mathbf{Z} = \boldsymbol{\mu} + \boldsymbol{GU},$$

where **G** can be obtained by Cholesky decomposition of the covariance matrix, $\boldsymbol{\Sigma} = \mathbf{G}^{T}\mathbf{G}$, $\boldsymbol{\mu}$ is the mean log-spectrum and **U** a standard normal random vector. Thus, we can easily simulate realizations of the target spectra. In order to obtain a set of hazard consistent ground motion, we do then have to simulate artificial accelerograms that match the simulated set of target spectra one-by-one. This can be achieved by making use of the classical so-called Random Vibration Theory (RVT) approach.

We consider the artificial ground motion time-histories as realizations of a zero-mean Gaussian process Y(t) defined by its evolutionary power spectral density (PSD). In this paper, we consider the case in which the modulating function does not depend on frequency, that is

$$S_Y(\omega, t) = q(t)^2 S_X(\omega), \quad (2)$$

together with the Gamma modulating function:

$$q(t) = \alpha_1 t^{(\alpha_2 - 1)} \exp(-\alpha_3 t), \; t \in [0, T_d] \; . \quad (3)$$

In order to comply with a target response spectrum, a "response-spectrum compatible" PSD S_X has to be identified. This is described in the following section.
The RVT allows to establish a relationship between the median pseudo-acceleration response spectrum $\hat{S}_a(\omega_n, \xi_0)$ and the variance, obtained as the integral of the power spectral density function of a stationary stochastic process, via the peak factor η. This result has been used by Vanmarcke (1976) to establish the inverse formula:

$$S_X(\omega_n) \approx \frac{1}{\omega_n(\frac{\pi}{2\xi_0} - 2)} \left[\frac{\hat{S}_a{}^2(\omega_n)}{\eta_T^2} - 2 \int_0^{\omega_n} S_X(\omega) d\omega \right]. \quad (4)$$

that allows to evaluate the PSD from a given response spectrum, for discrete positive frequencies ω_n. In the above expressions, ξ_0 designs the damping value used to determine the response spectrum. The simulation of time histories, given the respective PSD, is straightforward by means of the spectral representation theorem: $Y(t) = q(t) \int_{-\infty}^{\infty} e^{i\omega t} dZ(\omega)$, where $dZ(\omega)$ are the complex orthogonal increments associated to the stationary PSD.

The practical implementation of the algorithm is summarized in what follows. The following steps are repeated N times in order to obtain a database of N artificial accelerograms.

- Generate a realization of the correlated random log-spectral accelerations $\ln(S_a(T)), T = \{T_1, T_2, \dots T_{N_s}\}$ by using expression (1).
- Determine the median-spectrum compatible PSD $S_X(\omega_n)$ for frequencies $\omega_n = \{\omega_1, \omega_2, \dots \omega_{N_s}\}$ using formula (4).
- Generate a random realization of strong motion duration T_{SM} and the initial instant t_1. Both parameters are supposed to be log-normally distributed with given median value and log-standard deviation. Evaluate the parameters α_1, α_2 and α_3 of the modulating function. The normalizing constant is evaluated according to the equivalent energy criterion (equivalent stationary process).
- Construct the PSD of equation (2) and perform iterations in order to adapt the spectral content for best matching with the particular response spectrum considered.
- Simulate the artificial spectrum-compatible accelerogram by virtue of spectral representation.
- Apply the high pass filter proposed by Clough & Penzien (1993) and further used by Rezaeian & Der Kiureghian (2010) in order to obtain displacement time histories that do not diverge but yield zero (or constant) residual displacement.

More details can be found in the paper by Zentner (2014) and the *Code_Aster* online documentation. The Nelder Mead simplex search of Matlab is used here for the identification of the parameters of the modulating function that best fit the strong motion duration and.

Application

In this section, a number of comparative studies are performed in order to assess the latter methodology. For this purpose, the NGA database together with the Campbell & Bozorgnia (2008) attenuation model established during NGA West 1 project for 5% damping and for frequencies ranging from 0.1Hz-100Hz is considered.

Two kinds of comparisons are carried out

i. Comparison between a set of natural accelerograms, selected according to the Baker & Jayaram (2011) procedure, a set of simulated time histories and values predicted by the attenuation model.
ii. Comparison of the Ground Motion Indicators (GMI) of a set of artificial accelerograms, simulated in agreement with a conditional target spectrum, to the values predicted by the attenuation and correlation models.

In the two analyses, both median and standard deviation of the GMI are evaluated. The scenario considered is a strike slip type fault with dip angle 90°. For the first study, we consider (i): magnitude M=6.5 and distance R=20km event for hard soil with V_{s30}=760m/s; and for the second study (ii): magnitude M=6 and distance R=20km event for medium soil with V_{s30}=500m/s
For the stochastic simulations, the Gamma modulation function is used. The strong motion duration T_{SM} was modeled as a lognormal random variable with median $\hat{T}_{sm}$ and logarithmic standard deviation σ_{Tsm}. The scenario specific strong motion duration and its variability is chosen according to the relations found by Kempton & Steward (2006). The instant t1 (beginning of strong motion phase) is modeled as a uniform random variable taking its values in the interval [1s,5s]. In order to assure adequate space filling design, the two random variables are sampled according to the Latin Hypercube Sampling scheme (Helton et al. 2003).

Test case i): comparison between GMI of a set of artificial and natural, selected accelerograms and comparison to the GMPE

For the sample of simulated time histories, a sample of N=21 spectra has been chosen among a total of 200 simulated spectra so as to best fit the median and median +1 σ_{lnSa} spectra of the Campbell & Bozorgnia (C&B) GMPE. For the selection of the natural ground motion according to the Baker et al algorithm, the software provided by the authors, available for download on http://web.stanford.edu/~bakerjw/gm_selection.html, is used (case of unconditional selection for arbitrary components, no scaling). Since this paper has the purpose to assess the properties of artificial hazard consistent ground motion and compare them to natural time histories, the natural ground motion were not scaled to improve the match. It also has to be pointed out that the greedy algorithm, proposed by Jayaram et al. (2011) to improve the matching, was not used. This algorithm is based on the replacement of the selected time histories, one at a time, by another time history from the database and the check whether the spectral matching (in mean and standard deviation) is improved. It allows to improve the spectral match in the mean and the standard deviation but might introduce ground motion time histories not in agreement with the scenario, in terms of spectral shape which is not tested. Interestingly, this algorithm could also be implemented in the ground motion simulation procedure presented in this paper. In what follows, the properties of the simulated/selected ground motion time histories are compared to each other and to the reference from the GMPE. Moreover, the structural response to the Takeda oscillator is analyzed.
Table 1 presents a comparison between GMI of the two sets of simulated and selected (unconditional) scenario spectra. The response spectra of the two sets of selected (natural) and simulated ground motion time histories are shown in Figure 1 and Figure 2, respectively.

Table 1. Comparison of simulated (Zentner 2014), selected (Jayaram, Lin & Baker 2011) and predicted (Campbell & Bozorgnia 2008) GMI.

		Artificial simulated	Natural selected	GMPE (σ_{arb})
PGA g	Median log std	0.126 0.598	0.153 0.598	0.131 0.549
PGV m/s	Median log std	0.077 0.478	0.084 0.511	0.076 0.558
PGD m	Median log std	0.036 0.614	0.033 0.750	0.063 0.874
CAV gs	Median log std	0.377 0.595	0.370 0.393	0.338 0.429
PSA 2Hz g	Median log std	0.179 0.562	0.168 0.512	0.174 0.626
PSA 5Hz g	Median log std	0.330 0.624	0.299 0.595	0.332 0.618
ASA_{40} (2Hz) gs	Median log std	0.142 0.612	0.116 0.490	0.137 0.776

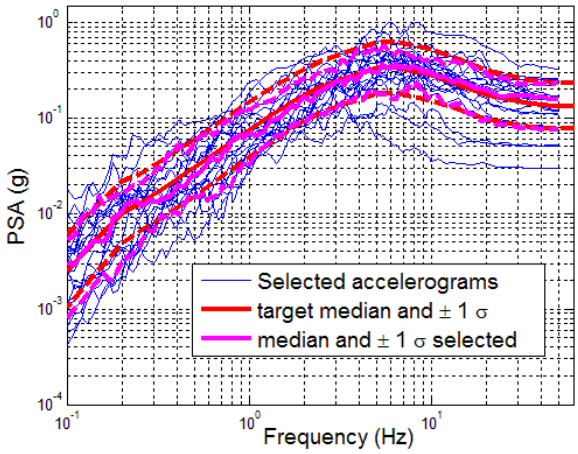

Figure 1. Response spectra of selected accelerograms using the Jayaram, Lin & Baker 2011 algorithm.

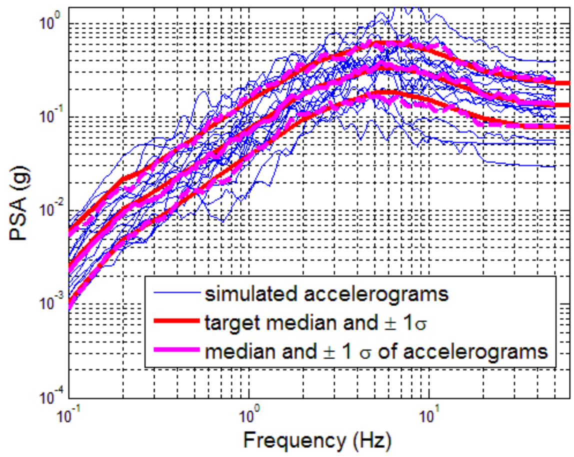

Figure 2. Response spectra of simulated accelerograms using the Zentner 2014 algorithm.

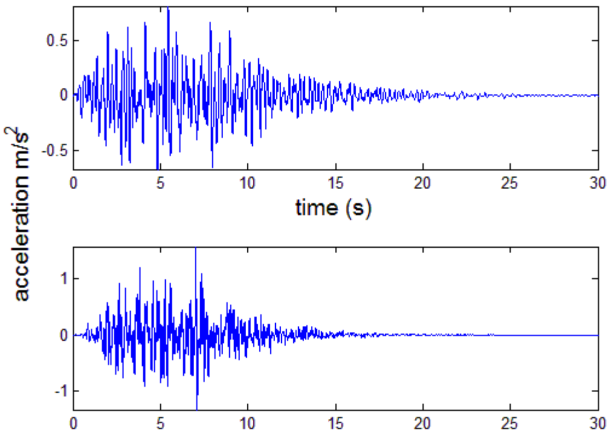

Figure 3. Two examples of simulated accelerograms.

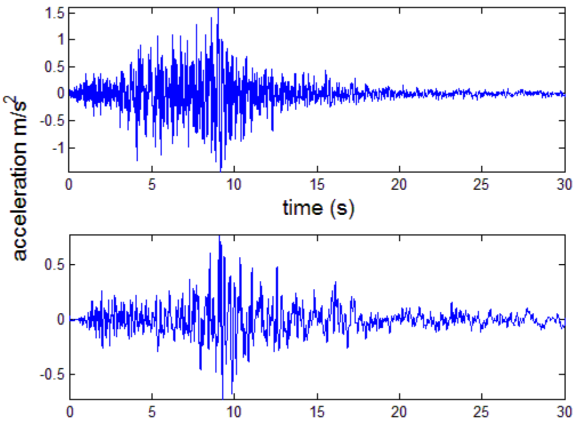

Figure 4. Two examples of selected accelerograms.

Table 2. Statistics of response of the Takeda oscillator: drift (m).

	artificial	natural
mean	0.0191	0.0208
median	0.0143	0.0146
std	0.0143	0.0162

Possible differences between the two sets of accelerograms are further analyzed by considering the structural response. For this application, a very simple model, the nonlinear Takeda oscillator with eigenfrequency 2Hz, viscous damping ratio ξ=0.05, strength reduction R=1 and Takeda model parameters α=0 and β= is chosen.
Table 2 provides the statistics of the response. The mean value, median, standard deviation (std) of drift, evaluated for the simulated and the natural accelerograms, are compared. The std of the natural accelerograms is slightly higher than the one of the simulated ones. In figure 5, the drift is plotted against the Pseudo Spectral Acceleration (PSA) evaluated at the oscillator frequency (2Hz).

It can be concluded that the simulated accelerograms are quite close to the target and to the selected natural accelerograms.

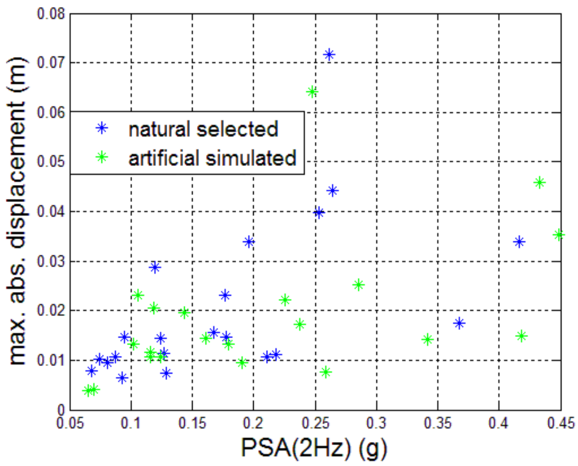

Figure 5. Scatterplots of max abs displacement against PSA(2HZ) for simulated (green) and selected (blue) accelerograms.

<u>Test case ii)</u>: comparison of GMI of artificial accelerograms to the predicted values (GMPE) when the target is a conditional spectrum

For illustration, we consider the case where the target value at 4.5 Hz is specified as the mean +1σ log-spectral acceleration at that frequency. This corresponds to ε=1 at that frequency, using the terminology used in Baker 2011. This yields a target design level $Sa(4.5Hz) = 0.6725g$. In consequence, the target ground motion parameters have to be updated. The conditional median and log-standard deviation of PGV and CAV can be obtained by virtue of formula (6) and (7) and using the empirical correlations determined by Wang & Du (2012) and Bradley (2012). The empirical correlation between the target spectral acceleration and the ground motion parameters CAV and PGV were taken as ρ=0.50 (according to Wang & Du 2012) and ρ=0.71 (according to Bradley 2012), respectively. The target values for PGA are taken from the CMS. These values are shown in Table 4 where they are compared to those of the simulated accelerograms. The comparison revealed good agreement except for the CAV which is overestimated. Figure 6 illustrates the matching between the spectra of the simulated accelerograms (blue) with the simulated target spectra (green). The target Conditional Mean Spectrum (CMS) and the target scenario spectrum are plotted in red for comparison.

Table 4. Conditional median and log-standard-deviation of GMI estimated from a sample of N=20 artificial accelerograms (ε=1), results from Zentner (2014).

Parameter	**PGA**	**(g)**	**PGV**	**(m/s)**	**CAV**	**(gs)**
	median	log std	median	log std	median	log std
C&B GMPE (2008, 2010); Wang & Du (2012) and Bradley (2012) correlation coefficients	0.247	0. 261	0.221	0.375	0.766	0.363
Artificial accelerograms, $T_{SM} \sim LN(\ln(\hat{T}_{SM}), \sigma_{TSM})$	0.240	0.267	0.231	0.479	0.903	0.307

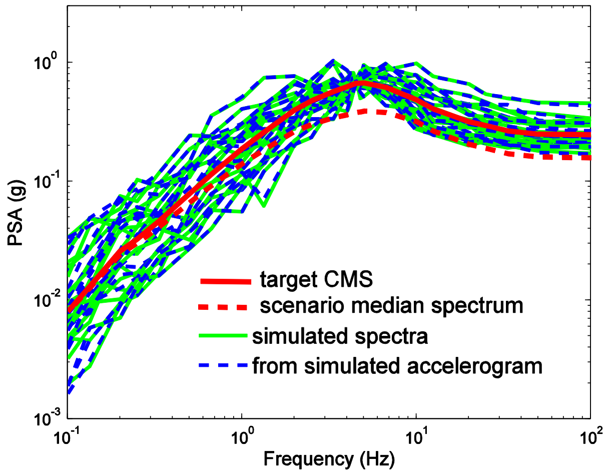

Figure 6. Response spectra of simulated accelerograms compared to target (red)

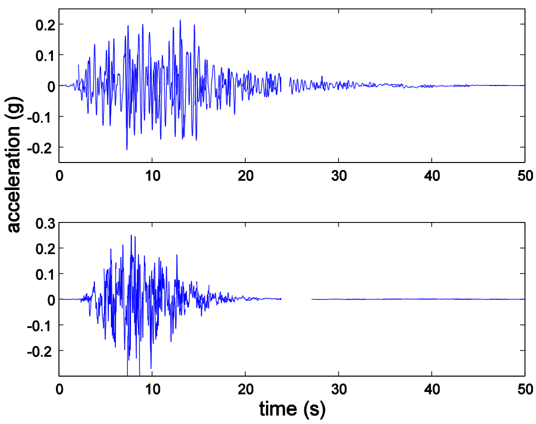

Figure 7. Two examples of simulated conditional accelerograms.

The ground motion selection procedure is limited by the number of accelerograms contained in the database (and matching the target). On the other hand, the simulated time histories do not feature an evolution of frequency content with time while the selected accelerograms posses all properties of natural ground motion.

Discussion and Conclusions

Ground motion selection or simulation is a topic largely discussed in the literature. The natural accelerograms possess the advantage of being related to real events, thus possess "realistic" non stationary features. However, due to the lack of data, the natural accelerograms are generally chosen from regions and site conditions different from the studied one, as long as they match the target spectral amplitudes. Moreover, scaling is often applied. Sometimes, the spectral content is also modified which results, in a strict sense, in artificial time histories. Artificial accelerograms can also be simulated by using a stochastic model which possesses the advantage that a great number of time histories can be obtained. This is particularly useful for probabilistic and reliability analysis. Besides, major properties of the accelerograms can be controlled and tuned during the simulation. In particular, the strong motion duration is a parameter of the stochastic ground motion simulation model and can be chosen in according to the considered scenario. Variability is accounted for by modelling

strong motion duration as a lognormal random variable, in agreement with recent GMPE (e.g. Kempton & Steward 2006). The strong motion duration cannot be controlled when spectral shape is the only criterion for ground motion selection. Concurrently, the simulated accelerograms have the advantage that as many accelerograms as desired can be obtained (for example for more advanced reliability and sensitivity analysis).
Jayaram et al. 2011 propose a greedy algorithm to improve the matching. Such an algorithm could also be used to improve the adjustment of the set of simulated accelerograms to the target spectra.

Further tests have to be performed in order to evaluate the adequacy of artificial ground motion. Also, the comparison between artificial and natural ground motion could be made more meaningful by using the same set of simulated target spectra for the analysis.

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