

## FLUIDE-STRUCTURE INTERACTION IN SEISMIC ANALYSIS OF ARCH DAMS

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**Abstract:** This paper aims at studying the dynamic behaviour of arch dams, taking into account Fluid-Structure Interaction (FSI). For this purpose, the couple Boundary Elements and Finite Elements (BE-FE) analysis is used to take into account the FSI. This method is firstly verified by an analytical solution in a case of simple model. The results obtained by the BE-FE model are then compared with the classical Westergaard approach (ref. 0) in two cases study. The influence of the compressible and incompressible fluid is also examined by using the BE-FE model.

### Introduction

The dynamic analysis of arch dams under seismic loadings is of main importance in the design process, since most of the arch dams are located in seismic areas. Generally, the arch dam body is modelled using the Finite Element Method. To address fluid-dam body interactions, it is necessary to take into account the reservoir in the model. Different methods can be applied to model the dam-reservoir system. Traditionally, due to difficulties in performing satisfactory analyses, the overall problem is solved by applying additional masses on the upstream face of dam. These additional masses represent the hydrodynamic pressure and are determined by Westergaard classical approach (ref. 0). However, Westergaard approach is generally conservative. In case of significant dam/reservoir interaction, this method may not be realistic enough to assess the dam response.

An alternative solution is the use of the BE-FE method to take into account the dam body flexibility as well as the radiation damping of the semi-unbounded reservoir. In this method, the dam body is modelled by FE method and the reservoir by BE method. The two methods are then coupled in a subspace of smaller dimension than the original FEM approximation subspace. Thus, this technique governs against a very high computational demand for modelling the 3D complex geometry of the reservoir.

### Seismic Fluid-Structure Interaction

Let's describe, herein, the fluid-structure interaction of an arch dam with a simple physical model, by spring-masse-damper system, as presented in figure 1. In this model, the dam body motion is represented with a single degree of freedom. During the earthquake, some part of the fluid in the reservoir follows the dam body motion, leading to create an additional mass. Thus, in the simple physical model, a mass can be added to represent this inertial effect of the fluid.

The semi-unbounded reservoir can be a way to generate the radiation, providing the energy dissipation for the new system. The additional mass is thus linked to the dam body by a simple spring and damper. The added spring associated with the additional mass may represent the fluid resonance in the reservoir and the damper may represent wave radiation into the reservoir.

It is obviously that the equivalent natural frequency of the fluid-structure system is smaller than the dam body one due to the inertial effect of the fluid. Furthermore, the fluid-structure system can have a new spectral response due to the damping effect of the reservoir.

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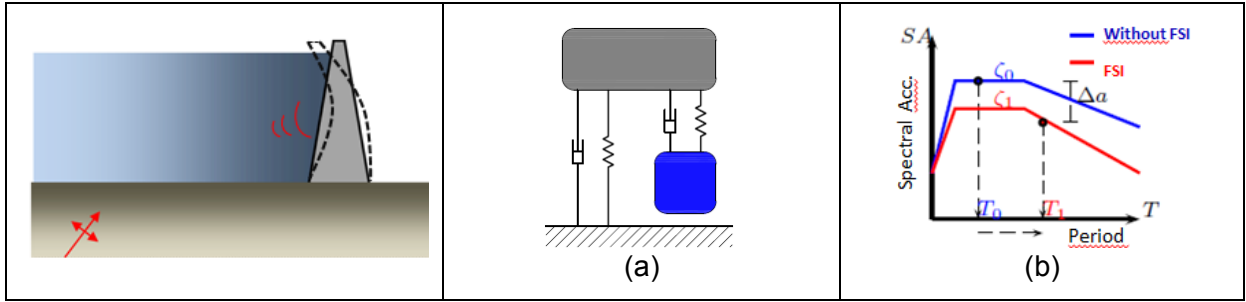


Figure 1. Seismic Fluid-Structure Interaction of an arch dam (a) Simple physical model and (b) Spectra response with and without FSI.

### Westergaard classical approach

H.M. Westergaard (ref. 0) studied the water pressure acting on dam during earthquake. The equations of motion are determined with the assumption of a perfect fluid (without viscosity) and small displacements. In order to find an analytical solution, boundary conditions are simplified: the dam is considered as a vertical rigid body, with a sinusoidal movement and the reservoir is semi-infinite. Under these conditions, the overpressure corresponds to a sum of sines:

$$p = -\frac{8\alpha\gamma_w h}{\pi^2} \cos\left(\frac{2\pi t}{T}\right) \sum_{n=1,3,5,\dots} \frac{1}{n^2 c_n} \sin\left(\frac{n\pi y}{2h}\right) \quad (1)$$

With :

$$c_n = \sqrt{1 - \frac{16\gamma_w h^2}{n^2 g K T^2}} = \sqrt{1 - \frac{1}{n^2} \left(\frac{T_0}{T}\right)^2} \quad (2)$$

$\alpha$  corresponds to the seismic coefficient (% g),  $T$  to the period of dam movement,  $y$  to dam face elevation ( $y=0$  at crest,  $y>0$ ),  $h$  to reservoir height,  $\gamma_w$  weight of water per unit volume,  $K$  to Bulk water modulus.

The analytical solution anticipates a resonance of reservoir – when  $c_n$  become null – around a  $T_0$  period, due to the water compressibility.

$$T_0 = \frac{4h}{c} \quad (3)$$

By neglecting the water compressibility, the overpressure becomes smaller than the well-known Westergaard parabola:

$$p \leq \frac{7}{8} \alpha \gamma_w \sqrt{y \cdot h} \quad (4)$$

On a structural point of view, this overpressure acts in similar way than a small water layer which remains solidary to the dam during earthquake. In finite element model the Westergaard classical approach consists in simulating the fluid-structure interactions by adding additional mass, determined by the previous equation (4). It has two effects on the structure: the overpressure is simulated by increasing the inertia loads, and the

eigenfrequencies are lower. As this method is directly applicable in structural software, it was commonly used last decades.

Main Westergaard approach assumptions are questionable: the water compressibility and resulting resonance of reservoir allow in rare cases – when the dam is really rigid – the added Westergaard masses are not conservative. But for most cases, this basic approach overestimates in large way the fluid-structure interaction, because on the one hand the dam flexibility amplifies the earthquake by a factor 5 to 10 at crest – and in a similar way the added masses inertia forces – and on the other hand the reservoir damping is neglected. Westergaard classical approach is successfully used for rigid and medium height dam subjected to moderate earthquake but it is not satisfactory for high dams or strong earthquakes.

### Coupling BE-FE Method

First, let's call the bounded volume of the dam body by  $\Omega_b$ , the semi-unbounded volume of fluid and soil by  $\Omega_f$  and  $\Omega_s$  respectively, and the interfaces between the sub-domains by  $\Sigma_{fs}$ ,  $\Sigma_{fb}$  and  $\Sigma_{bs}$  as shown in figure 2. Therefore, "s" stands for the soil, "f" for the fluid and "b" for the dam body. The permanent displacement due to static loads (the weight or the hydrostatic pressure) is assumed to be known in the following. Therefore, the aim of this work is to determine the dynamic perturbations due to seismic loading. It should be mentioned that the dynamic perturbation is assumed to be small enough to lead to small strain tensor in each domain.

The dam body is supposed to be linear and elastic and the fluid is either compressible or incompressible, assuming that the flow is irrotational. The rock foundation is supposed to be massless to ignore any incident wave propagation and radiation in this sub-domain.

The pressure field "p" satisfies the wave equation inside  $\Omega_f$  and the free surface conditions:

$$c^2 \Delta p - \partial_{tt} p = 0 \text{ inside } \Omega_f \quad (5)$$

$$p = 0 \text{ on free surface} \quad (6)$$

where the acoustic wave velocity "c" is a function of the compressibility and the density of the fluid  $\rho_f$ , while  $c=0$  for the incompressible fluid. The displacement and the pressure fields satisfy the following local equilibrium conditions on the interface  $\Sigma_{fb}$ :

$$\rho_f \partial_{tt} u_b \cdot n + \partial_n p = 0 \quad (7)$$

$$t_b(u_b) - p \cdot n = 0 \quad (8)$$

Where  $u_b$  is the dynamic displacement field in  $\Omega_b$  and  $t_b$  is the traction vector and  $n$  is the unitary normal vector on  $\Sigma_{fb}$ .

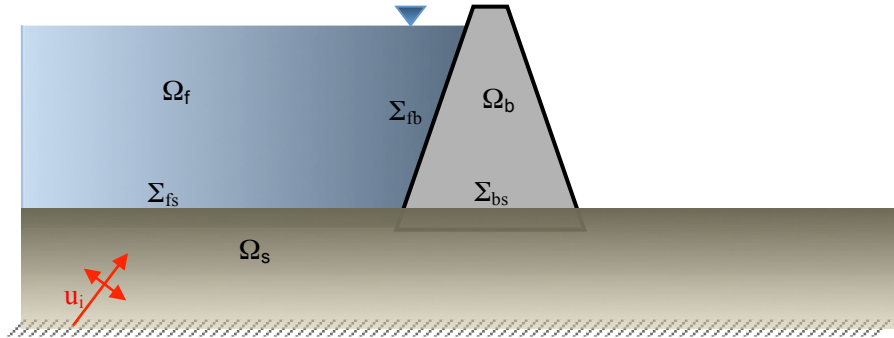


Figure 2. Harmonic frequency response of dam at crest taking into account the FSI.

Sub-structuring is commonly used to study the problems of the interaction between the different sub-domains. Herein, this technique is applied for the fluid-structure system subjected to seismic loading. In order to reduce the size of the problem, the finite element method based on modal analysis is used to solve the global problem in a reduced system. Thus, the displacement field in the dam body can be decomposed into two modal bases: the rigid body modal basis due to the rigid translation of the basis of the dam and the flexible modal basis of the dam body with fixed base conditions. Thanks to this decomposition, the global problem size can be reduced to  $3+20=23$  modes, in which 3 corresponds to the three translation modes and 20 (generally) corresponds to the first 20 eigenmodes as

$$\mathbf{u}_b = \sum_{k=1}^{3\text{-modes}} c_k(\omega) \mathbf{l}_{bk}(x) + \sum_{m=1}^{20\text{-modes}} q_m(\omega) \phi_{bm}(x) = [\mathbf{c} \quad \mathbf{q}] \begin{Bmatrix} \mathbf{L} \\ \Phi \end{Bmatrix} \quad (9)$$

Where  $\mathbf{l}_{bk}$  and  $\phi_{bm}$  are the rigid body modes (three in translation) and the eigenmodes (the first 20 dynamic modes), respectively, and “ $\mathbf{c}$ ” and “ $\mathbf{q}$ ” are the vectors of the associated modal coordinates. Based on this decomposition, the equation of the motion in the frequency domain for the couple dynamic fluid-dam system can be written as

$$\left( \begin{bmatrix} Z_{11}^f(\omega) & Z_{12}^f(\omega) \\ Z_{21}^f(\omega) & Z_{22}^f(\omega) \end{bmatrix} + (1 + 2i\beta) \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{A}_{22}^b \end{bmatrix} - \omega^2 \begin{bmatrix} M_{11}^b & M_{12}^b \\ M_{21}^b & I_{22}^b \end{bmatrix} \right) \begin{bmatrix} \mathbf{c}(\omega) \\ \mathbf{q}_b(\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{eq}(\omega) \\ 0 \end{bmatrix} \quad (10)$$

where the impedance of the fluid  $Z^f$  and the equivalent seismic force  $\mathbf{f}_{eq}$  are computed by BE and the FE method is applied to determine the mass and stiffness matrices projected to the modal basis. In the above equation, the diagonal matrix  $\mathbf{A}_{22}^b = \text{diag}(\omega_j^2)$  contains the square of the fundamental circular frequencies of the dam body with fixed base and  $I_{22}^b$  is the identity matrix reflecting the orthonormality of the eigenmodes with respect to the mass matrix of the dam body.  $M_{11}^b$  and  $M_{12}^b$  are matrices as well, including respectively, the total mass and the modal participation factors of the dam body. It should be noted that in above equation the global matrices are divided into two sub-blocks to represent the generalized stiffness and mass matrices of the translation modes by sub-block 1 and of the eigenmodes by sub-block 2. In order to present the response in time domain the frequency variable  $\omega$  is transformed to time “ $t$ ” by means of an Inverse Fourier Transform Function.

It is worth to notice that this numerical BE-FE method has been implemented in MISS3D code which is developed at Ecole Centrale Paris in collaboration with Tractebel Engineering (France) - Coyne et Bellier.

This numerical model is then verified by the Westergaard analytical solution in case of a rigid dam body (figure 3).

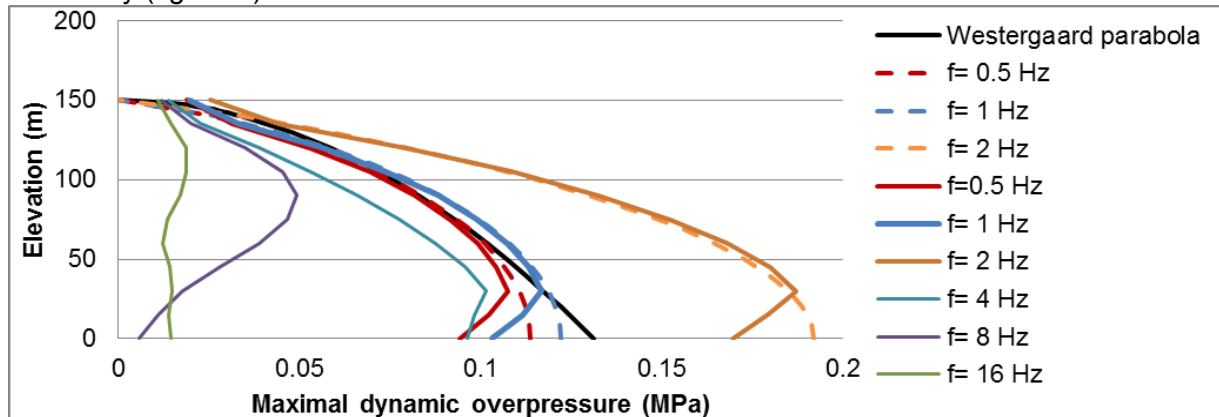


Figure 3. water overpressures acting on dam – comparison between analytical solution and coupled FE-BE method. The water compressibility introduces a reservoir resonance around 2 Hz.

### Case Study 1

The Westergaard classical approach and the seismic FSI were firstly compared on a 220 meters high arch dam.

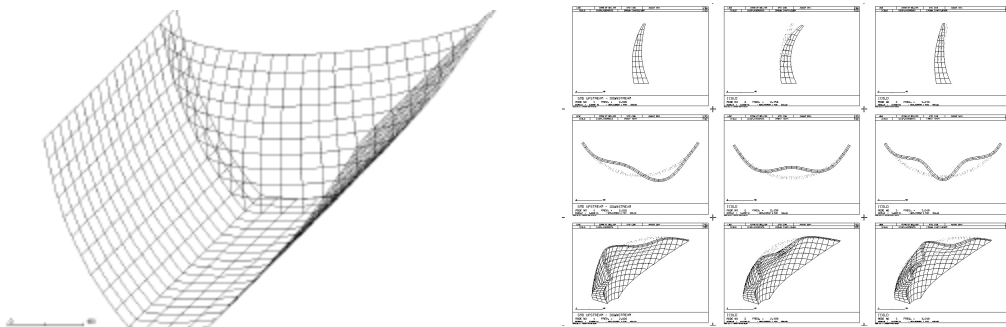


Figure 4. dam and reservoir mesh. The BE element method is an elegant way to solve semi-infinite problem. Only the interface shall be meshed. The dam behaviour (impedance) is determined with a traditional finite element software

Five analyses were performed: in first time, no fluid-structure interaction is considered (empty reservoir). The full reservoir is introduced with respectively a rigid and flexible dam, both of them analysed with the Westergaard approach and a BE-FE coupling model.

The earthquake is amplified by a factor 8 at crest. Crest displacements are 1.5 as important as results with an empty reservoir (4 cm).

By considering the dam as a rigid body, both approaches have similar results. It means for this case the reservoir resonance can be neglected.

However by introducing the dam flexibility, the Westergaard approach overestimates the waterpressure acting on dam by a factor two: the corresponding pressures are deduced by multiplying the maximal acceleration on dam face (increasing with the dam height) by the additional masses (decreasing with the dam height). The Westergaard approach is not very representative for flexible structure.

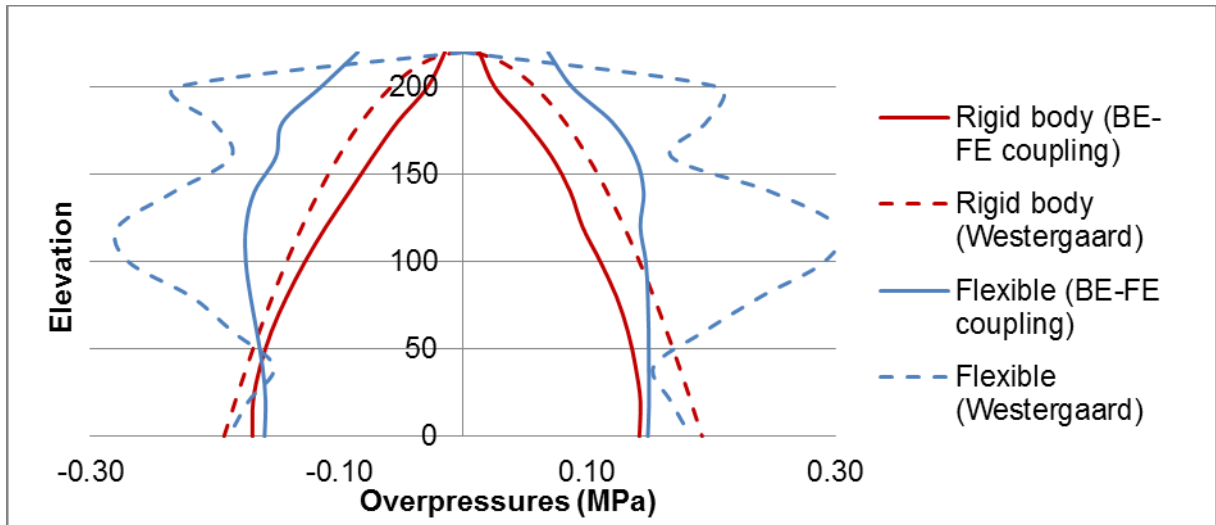


Figure 5. maximal water overpressures – comparison between Westergaard approach to the BE-FE coupling

### Case Study 2

An arch dam under construction is chosen to study the seismic FSI. This dam is located in a seismic area, in Turkey, with the PGA of  $3m/s^2$ . The dam stands 181m high and 220m long at its crest. The thickness of the crown cantilever varies from 56m at the base to 8m at the crest. This dam provides a hydraulic head of 166 m to produce a capacity of 332 megawatts. The concrete in the dam body is assumed to be linearly elastic, with the following properties: density= $2.35t/m^3$ ; Poisson's ratio = 0.2; Young's modulus = 20GPa; and a hysteretic damping factor of 5%. The foundation rock is assumed to be massless with the Young's modulus=10GPa and Poisson's ratio=0.2. The density of water is  $1t/m^3$  and the wave velocity in compressible fluid is assumed to be 1440m/s.

The FE model of the dam and the foundation is presented in figure below. The solid elements are used to model the dam body and the rock foundation. The BE method is used for the semi-unbounded reservoir domain by using the quadrilateral elements introduced on the upstream water-dam interface as well as the reservoir-rock. The BE model is truncated at the location about one time of water depth.

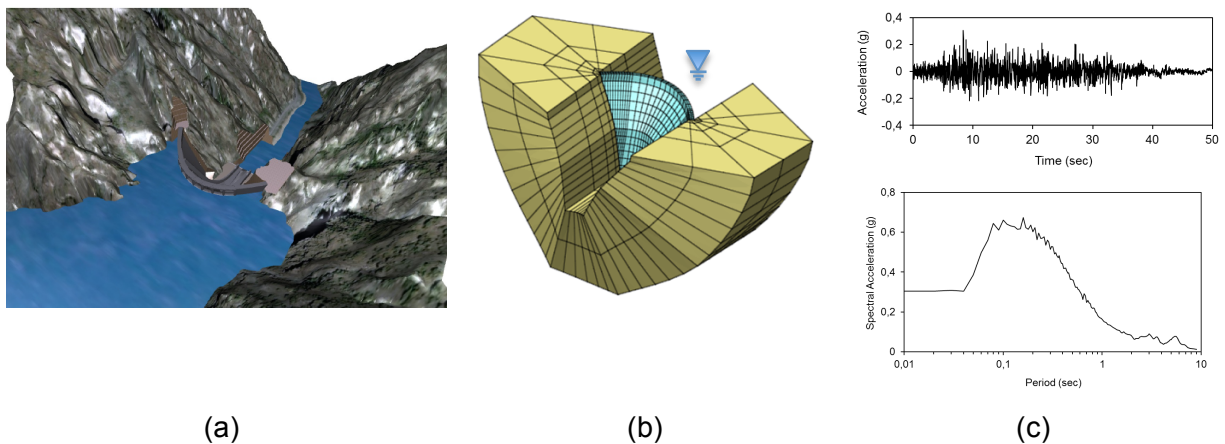


Figure 6. (a) Upstream view of the reservoir-dam body; (b) Numerical model of the arch dam with its massless foundation; (c) Ground acceleration and 5%-damped response spectra.

Let's start with the impedance of the fluid as the first important parameter for the couple FSI analysis. Figure 7 presents the real part and the imaginary part of the impedance for both compressible and incompressible fluid. It can be seen that the impedance is strongly

dependent on the frequency. The real part increases parabolically with frequency, in the case of the incompressible fluid, showing the inertial effect of the fluid ( $-\omega^2 m$ ).

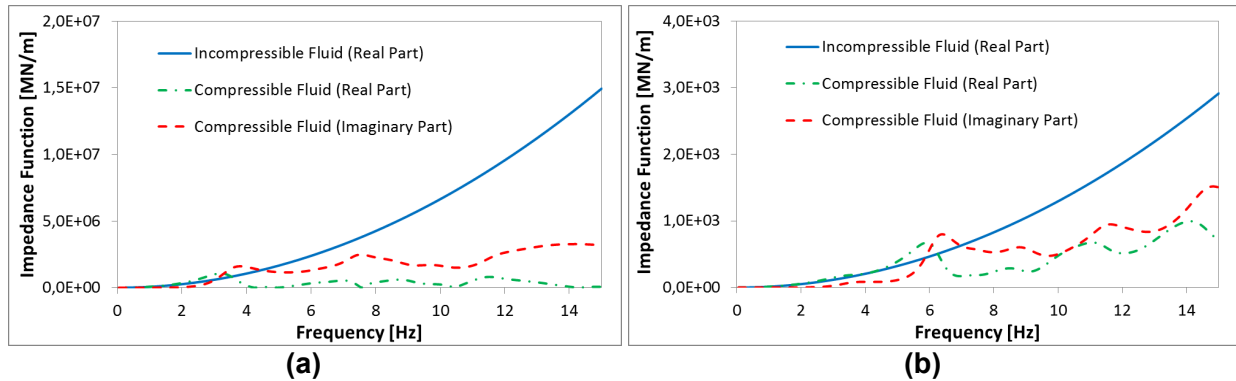


Figure 7. Impedance of the fluid computed by the couple BE-FE model for (a) the horizontal model and c) the first eigenmode.

As far as the compressible fluid is concerned, it is observed that the real part and the imaginary part increase with the frequency, exhibiting the fluctuation behaviour. Indeed, this fluctuation may be due to the fluid resonance in the reservoir. The imaginary part of the impedance represents the radiation damping due to wave propagation into the semi-unbounded reservoir.

Figure 8 presents the harmonic frequency response of dam at crest, computed by the couple FSI analysis. The FSI system creates the new resonances taking place at frequencies lower than the one of the dam body with the fixed base. This is due to the fact that the fluid is considered as an additional mass for the dam body. Although the fundamental frequencies of the couple FSI system remain the same for both compressible and incompressible fluid, it is observed that the compressible fluid reduces strongly the dam response thanks to the energy absorption by the additional damping due to the wave radiation into the semi-unbounded reservoir.

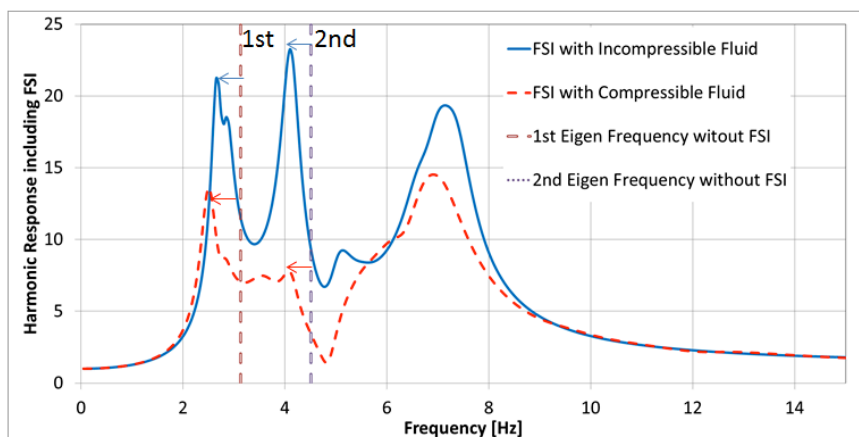


Figure 8. Harmonic frequency response of dam at crest with taking into account the FSI.

The same remark can be observed in time domain relative displacement of the dam body. As shown in figure below, a comparison between compressible and incompressible fluid shows that the compressible fluid reduces significantly the dam response, confirming the damping of the compressible fluid.



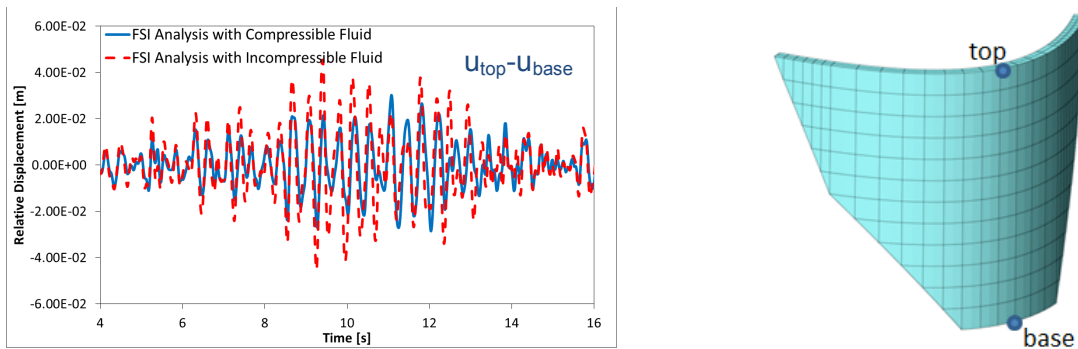


Figure 9. Comparison between compressible and incompressible fluid based on the BE-FE model; Relative displacement time history response between top and bottom of the dam at the crown including fluid-structure interaction

It is now interesting to compare the couple BE-FE model with the Westergaard approach. It can be clearly seen that the Westergaard approach overestimates significantly the dam response, showing that this approach is very conservative in the earthquake design of the case study arch dam.

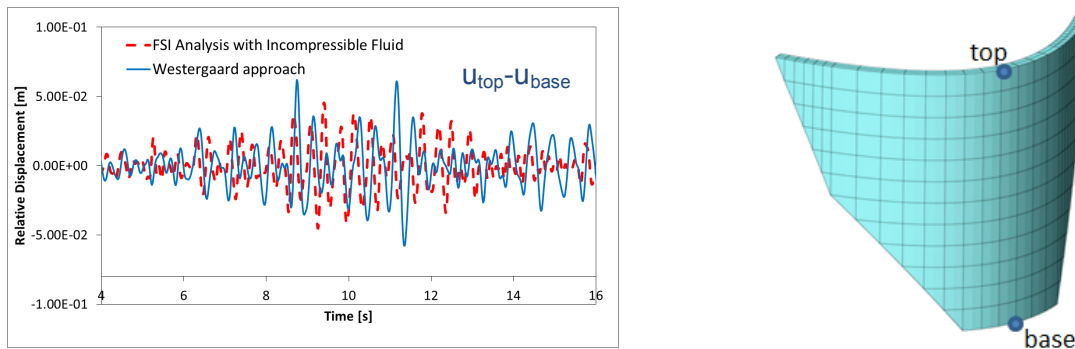


Figure 10. Comparison between Westergaard approach and the BE-FE model; Relative displacement time history response between top and bottom of the dam at the crown including fluid-structure interaction

### Conclusion

This work applies an efficient numerical method based on the couple BE-FE model to study the dynamic behaviour of the arch dam. This method is very fast to determine the dam response, particularly, for a 3D complex geometry of the reservoir. In this paper, the numerical BE-FE model was verified by mean of an analytical solution in case of a simple model. Based on the results obtained by the BE-FE model, it was observed that the fluid compressibility decreases strongly the dynamic response of the dam due to the wave propagation into the semi-unbounded reservoir. The comparison between the Westergaard approach and the couple BE-FE model shows that the Westergaard approach is very conservative in earthquake design of the cases study high arch dams. On other words, using the FSI analysis by the couple BE-FE model can be a mean to optimise an arch dam project located in a seismic area.

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