A NOVEL APPROACH FOR THE CALCULATION AND PRESENTATION OF SECONDARY RESPONSE SPECTRA

Christopher PEARCE[1] & Ian WARD[2]

Abstract: *"Secondary response" describes the behaviour of secondary systems inside a building subjected to dynamic loading. The design of secondary systems, such as equipment, stairs, or other non-structural components, can be governed by these loads. Consequently, secondary response is an important consideration in the seismic design of structures. Since it is rarely feasible to employ detailed models of equipment arrangements throughout structural models to determine their dynamic behaviour, the design of equipment and other secondary systems are often based on Secondary Response Spectra (SRS). An SRS details the maximum response of a series of simple oscillators with different dynamic properties.*
SRS are typically calculated at a limited number of positions within a structure and then enveloped to produce bounding spectra. Inherent in this approach is the potential for the resulting SRS to be either overconservative, as the spatial resolution of the secondary response calculations is lost during enveloping, or to be nonconservative by missing peak values.
In this study, a novel "global SRS calculation and mapping" procedure is proposed, wherein SRS are calculated at all nodes within a finite element model of a structure. This approach has only recently become possible through the confluence of advances in computing power and efficient in-house solutions to the underlying mathematics. By preserving the resolution of the SRS calculations, this approach will avoid undue conservatism. It is amenable to automation, and can significantly reduce the potential for user error. In addition, with SRS data available at all points, the variation in secondary response throughout a structure can be communicated in new and intuitive ways.

Introduction

Secondary response describes the behaviour of non-structural components within a building subjected to dynamic loading. Secondary response is most often considered in the design of seismically qualified structures, but may also be considered in other scenarios such as ground-borne vibration. When a structure is subjected to an earthquake, it will experience acceleration at its base, caused by motion of the ground. The structure, or "primary system", will respond dynamically, which will induce accelerations acting on the "secondary system" housed inside it. The secondary systems considered are most often equipment, or items such as stairs, walkways etc. The objective of secondary response calculations is typically to obtain the peak accelerations experienced by the secondary systems during the dynamic event. The load generated by this peak acceleration will inform the specification or design of equipment, and any support structures or frames.

The most common method for determining secondary response is known as the "cascade approach" (Taghavi and Miranda 2008). In this approach, the first step is to determine the acceleration time-history at a point in a structure, as a result of the application of a base acceleration time-history. The point selected is typically on a floor slab on which equipment will be located, and the base acceleration time-history may represent seismic ground motion. With knowledge of the seismic design acceleration time-history, the response at any point in the structure can be determined through numerical or analytical models, such as the Finite Element (FE) method. The second step is then to determine the acceleration time-history of the equipment as a result of the structural acceleration. The fundamental assumptions of this approach are that the secondary system does not interact with the primary system, and that the dynamic properties of the primary system are not affected by the presence of the secondary system. These

[1] Chartered Engineer, Atkins (Member of the SNC-Lavalin Group), Epsom, United Kingdom, chris.pearce@atkinsglobal.com
[2] Technical Director, Atkins (Member of the SNC-Lavalin Group), Epsom, United Kingdom, ian.ward@atkinsglobal.com

assumptions are reasonable in the majority of circumstances, except when the primary system is expected to respond inelastically, or the secondary system is of substantial mass (Taghavi and Miranda 2008).

It is generally not feasible to model the equipment arrangements inside a structure in any great detail. The equipment is therefore typically modelled using a Single Degree of Freedom (SDOF) approximation. If the natural frequency and damping ratio of the piece of equipment is know or assumed, it can be represented by a SDOF system with equivalent properties. The response of an SDOF system to acceleration at its base can be readily obtained using numerical methods. Chopra (1995) describes several numerical methods available to calculate the behaviour of SDOF systems in response to discretised acceleration time-histories, such as the central difference method, and variants of Newmark's method. Of particular note is the recursive function developed by Nigam and Jennings (1969), which yields an exact solution for the behaviour of an SDOF system in response to an applied acceleration time-history assumed to vary linearly between discretised points. Guidance is available in codes such as ASCE 4-16 and ETC-C:2010 on the applicability of various numerical methods, based on the time-incrementation of the ground motion.

The dynamic properties of non-structural components, such as complex equipment arrangements, may not be known to the building designer, or may be subject to future alteration. As such, rather than determine the response of a single SDOF system, it is generally required to be knowledgeable of the peak acceleration response arising from a range of SDOF systems. This information is conveyed as Secondary Response Spectra (SRS). An SRS describes the peak response of a series of SDOF systems with different dynamic properties. Example SRS, showing peak acceleration against the natural frequency for different levels of equipment damping, are shown in Figure 1. The production of SRS is often required during the design stage or to inform remediation measures post-construction.

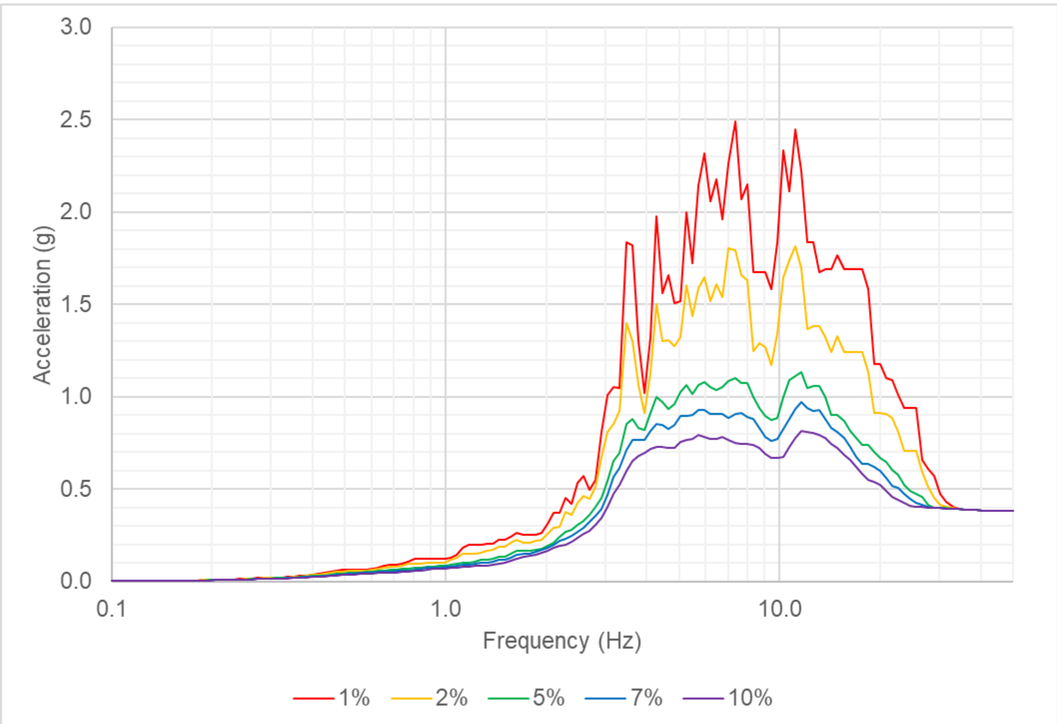

Figure 1. Example SRS plot detailing response of non-structural components vs. natural frequency. The various spectra correspond to different levels of equipment damping (from 1% to 10%).

Existing method

In practice, it is necessary to characterise the secondary response for large areas of a structure, rather than at individual points. In order to do so, the standard procedure is to calculate SRS at a limited number of positions within the structure, which are then enveloped to produce bounding spectra. For example, in order to determine SRS characteristic of a floor in a structure the following procedure would be followed:

1. First determine suitable points on the floor that would capture the maximum secondary response. This typically requires additional analysis to predict the locations where maximum response would occur, or manual steps to select relevant points based on engineering judgement.
2. Calculate SRS at the selected points.
3. Envelope the resulting spectra to produce a bounding spectrum. The bounding spectrum represents the maximum secondary response of the area analysed.

The existing method is depicted in Figure 2 for an example floor. The enveloping of five SRS is depicted, although in practice many more may be used.

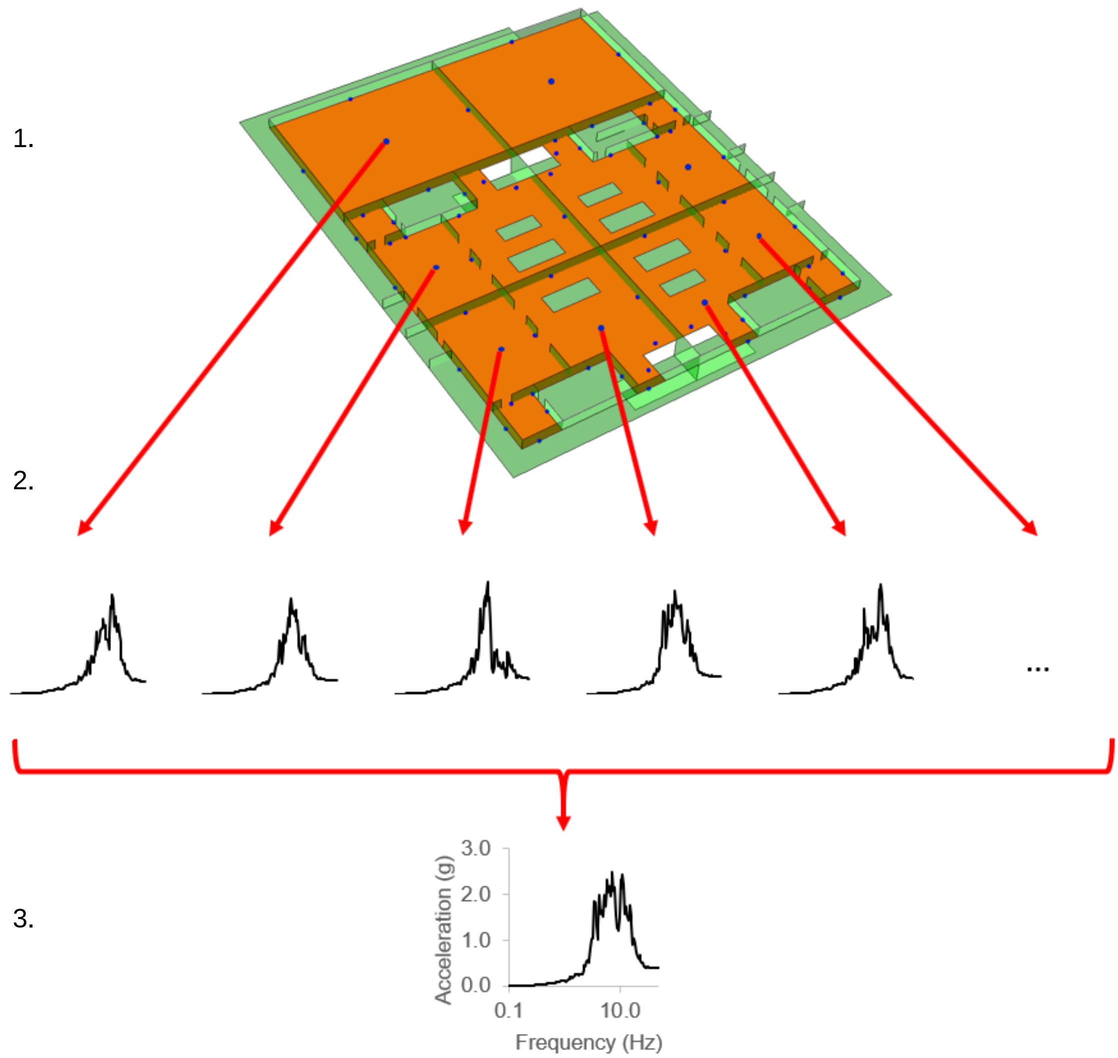

Figure 2. Diagram of existing SRS calculation procedure. To produce SRS characteristic of an example floor (highlighted orange), SRS are first determined at selected points (blue dots), and then enveloped.

Proposed method

Analysis philosophy

In this study, a novel approach to secondary response analysis is proposed: the "global SRS calculation and mapping" procedure, which is based on the calculation of SRS at all nodes within a FE model of a structure. This yields a number of benefits over the existing method:

- **Reduced conservatism / more control of conservatisms**
  In the existing method, the enveloped SRS determined for a particular region within a structure are assumed to be characteristic of the entire region. However, the response in some areas will be less onerous. In this study, it is proposed that SRS be calculated at all nodes within a model. Doing so removes the need to envelope SRS calculated at a limited number of points in order to characterise larger regions.

- **Fewer calculation steps**
  In the existing method, several additional calculation steps are required to determine the positions in the structure at which SRS are to be calculated. Defining SRS areas, or identifying and selecting positions that will give the greatest response, can be time-consuming manual steps. By comparison, in the global SRS calculation method, there is no longer a need for these additional steps.

- **High resolution**
  Despite the significant computational effort underlying the production of SRS, in the current method the results are communicated in a relatively coarse fashion, by way of a few SRS plots per region. The resolution of the results, present in the calculations themselves, is therefore lost.
  However, as a result of the proposed method, a higher density of SRS data is available, which allows the results to be communicated in novel ways. SRS results can be mapped onto computational models of structures and presented as contour plots. These contour plots could potentially be generated in any scriptable modelling software that would yield the most benefit for the analysis or design task at hand: for example, FE software such as Abaqus (Dassault Systèmes SE) or Robot Structural Analysis Professional (Autodesk Inc.) could be used, or Computer Aided Design (CAD) packages such as AutoCAD (Autodesk Inc.). This could benefit the positioning of equipment or other design processes that rely on SRS results, and would give designers and clients a more intuitive understanding of the behaviour of the structure and secondary systems.

Complimentary advancements

SRS calculations are computationally intensive, and as such, this approach has only recently become practical, due to advances in computing power. Enabling technologies include the proliferation of analysis computers with many high-speed processors, multi-processing software libraries, and the availability of systems with large amounts of Random-Access Memory (RAM).

Efficient solutions to the underlying mathematics have also been employed to facilitate the global SRS calculation method. A variant of the modal dynamic time-history analysis technique has been developed that significantly decreases the number of times the equation of motion of an SDOF system needs to be solved. This is achieved by substitution of the function for the acceleration response of the primary system within that of the secondary system, resulting in a direct response function for the secondary system. This allows factoring by appropriate modal properties (i.e. modal displacements and mass participation factors) to be detached from the solution of the SDOF equations of motion, resulting in significant time savings when computing SRS at a large number of FE nodes.

Case study

Software implementation

In-house Atkins software has been developed to implement the proposed global SRS calculation and mapping procedure. The software (SpectroScope) has been written in the Python programming language, and makes use of parallel processing, efficient array operations, and the other complimentary advancements described above in order to efficiently calculate secondary response. The exact solution to the equation of motion of SDOF systems is obtained using the recursive function presented in Nigam and Jennings (1969).

Analysis example

The above-mentioned software has made it feasible to calculate equipment response at all points in computational models of floors, walls, or entire structures. This has enabled a detailed understanding of the secondary response both over wide areas, or at individual equipment locations. It has been successfully employed on projects to determined secondary response for a wide range of structural models, of varying levels of complexity, and created in various software packages.

A case study is presented, demonstrating the global SRS calculation and mapping procedure conducted using a detailed FE model of a large, seismically qualified structure. A FE model of the complex, partially-embedded, reinforced concrete building was created using the Abaqus/CAE software package (Dassault Systèmes SE). The model was composed of approximately 360,000 shell and beam elements, defined by approximately 350,000 nodes, with around 230,000 spring elements representing the soil.

A modal analysis was conducted to obtain the information required for the secondary response calculations, namely: the natural frequencies, damping ratios (determined using composite modal damping), and mass participation factors in the three Cartesian directions, for all modes of interest. A total of 812 modes were considered, which represents all natural modes up to 40 Hz. In addition, the modal displacements in the three Cartesian directions, corresponding to each of the modes of interest, were determined for all nodes in the model.

The SpectroScope software was used to calculate both the structural and equipment responses to a set of concurrent base acceleration time-histories, (i.e. time-histories in the *x*, *y*, and *z* directions, representing components of seismic ground motion). SRS were determined for a wide range of dynamic properties of the secondary system: specifically, seven equipment damping ratios between 1% and 15%, and 150 equipment natural frequencies between 0.1 Hz and 50 Hz. The time required to calculate SRS is dependent on many factors, including the analysis machine used, the size of the model, and the number of equipment damping ratios and natural frequencies considered. In this case study, the calculation time to determine the SRS for each equipment damping ratio was approximately equal to the time required to conduct a modal analysis of the structure.

A variety of contour plots were then produced, utilising the SRS results determined at all nodes within the model. The large quantity of data can be visualised in different ways, but are here presented as contour plots of either: SRS at a specific equipment frequency, the peak SRS enveloped across all 150 equipment frequencies, or the equipment frequencies that give rise to the maximum SRS. In each case, the plotted results are specific to a single value of equipment damping. A few example contour plots are given in Figures 3 to 6, below.

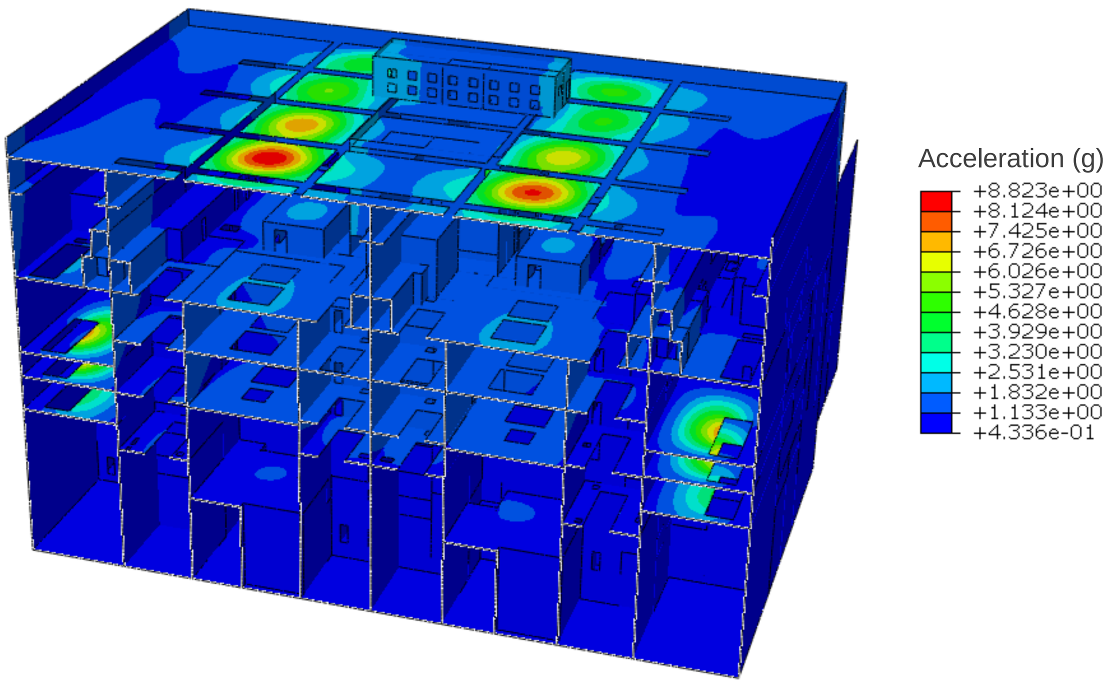

Figure 3. Contour plot of vertical (z-direction) SRS, computed for a specific equipment frequency of 10.2 Hz, with 5% equipment damping.

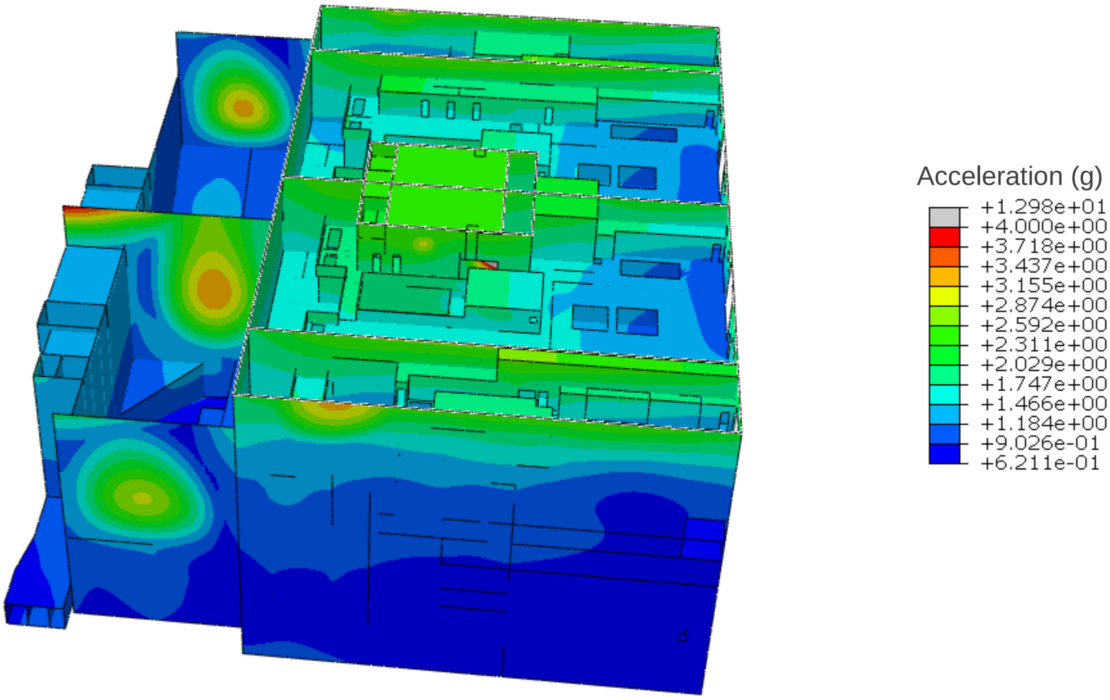

Figure 4. Contour plot of maximum horizontal (x-direction) SRS, enveloped across 150 frequencies between 0.1 Hz and 50 Hz, with 5% equipment damping.

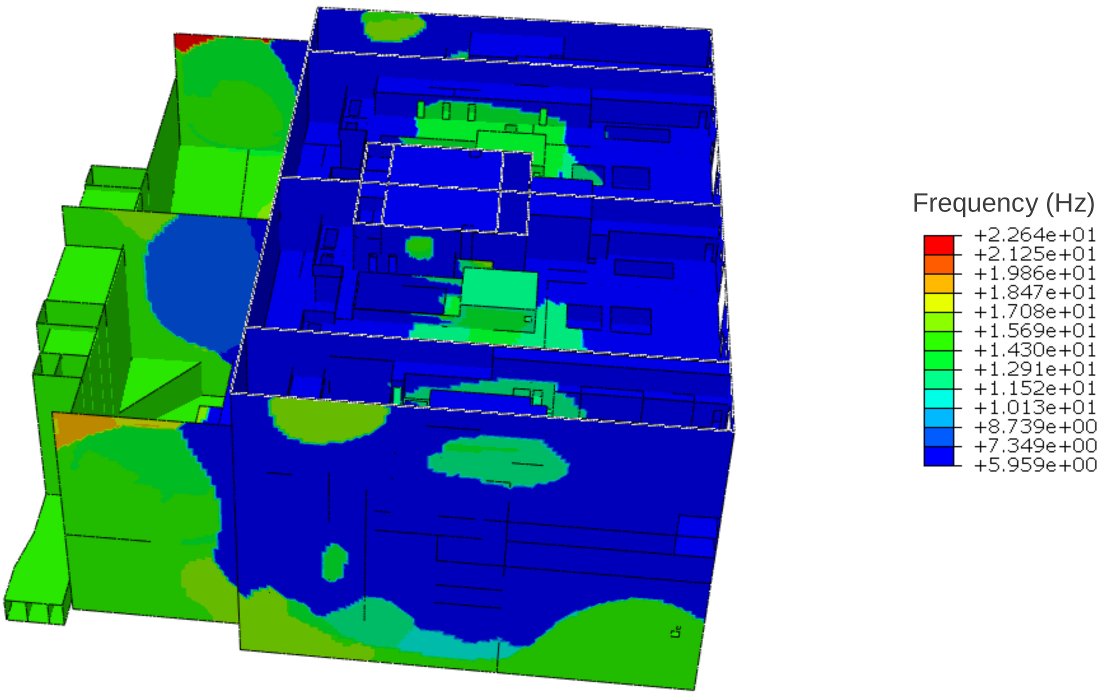

Figure 5. Contour plot showing equipment frequencies corresponding to the maximum horizontal (x-direction) SRS, enveloped across 150 frequencies between 0.1Hz and 50.0Hz, with 5% equipment damping.

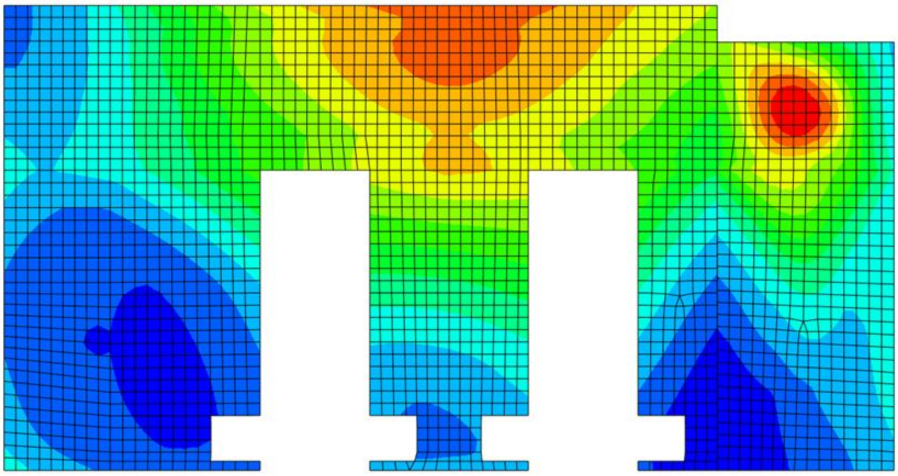

Figure 6. Contour plot of maximum vertical (z-direction) SRS, enveloped across 150 frequencies between 0.1 Hz and 50 Hz, with 5% equipment damping. Plan view of an individual floor, with FE mesh visible.

Discussion

There is a substantial amount of literature on methods to expediate the process of calculating SRS. Various "spectrum-to-spectrum" methods have been developed, in which SRS are determined analytically from the ground spectrum of the design earthquake. Several different techniques have been employed in developing these methods. For example, Yasui et al. (1993) and Jiang et al. (2015) describe spectrum-to-spectrum methods based on Duhamel's integral. Calvi and Sullivan (2014) present a spectrum-to-spectrum method that makes use of an empirical relationship proposed by Sullivan et al. (2013), which derives from a large number of non-linear time-history analysis simulations. In Lucchini et al. (2016), a spectrum-to-spectrum method based on a probabilistic approach is presented, that allows SRS to be determined for a target mean annual frequency of exceedance. Reviews of the currently available spectrum-to-spectrum methods can be found in Lucchini et al. (2017), and the aforementioned publications. Although a number of spectrum-to-spectrum methods have been available for some time, they have not been widely adopted in the nuclear industry. This may be due to the fact that the approximations inherent in several of these methods result in varying levels of conservatism over different frequency ranges, which could produce overconservative or nonconservative results at certain frequencies (Jiang et al. 2015).

One of the principal advantages of spectrum-to-spectrum methods is the speed at which SRS can be determined. The other is the fact that generating SRS directly from the ground spectrum overcomes the variabilities in SRS derived from spectrum-compatible time-histories, which generally results in multiple base acceleration time-history sets being required during design calculations. The computationally efficient time-history method discussed here has made it feasible to calculate SRS at all nodes in a structure. Nevertheless, spectrum-to-spectrum methods could also be employed in the global SRS calculation and mapping procedure if such a method were preferable.

There appears to be little literature concerning the selection of nodes for SRS calculations, or methods by which a greater understanding of the variation in secondary response throughout a structure can be obtained. A notable exception is Jussila et al. (2016), in which the authors conclude that the selection of nodes to be used for SRS calculations should not be based on engineering judgement alone, as this has the potential to be nonconservative. The authors then explore a systematic approach of selecting nodes in regular grid patterns of increasing density, in order to provide recommendations on the grid size to use for SRS calculations in reactor buildings. The authors note that predicted SRS may vary significantly within a floor, and in their study report a typical coefficient of variance of 0.2 for SRS in the horizontal directions and 0.45 for vertical SRS (Jussila et al. 2016).

The procedure presented in this study is in many ways similar to the philosophy of Jussila et al. (2016), which note that previous numerical methods were limited to computationally efficient simplifications, whereas modern methods can take advantage of the increased computation capacity available. The global SRS calculation procedure, wherein SRS are determined at all nodes, could be regarded as a logical extension to grid-based methods of increasing resolution. However, unlike grid-based methods, no additional calculation steps are required to identify the nodes of interest. Furthermore, the results of the global SRS calculation procedure are amenable to contour plot production, which leads to an intuitive understanding of the variation in secondary response within the structure.

Conclusions

Design procedures should reflect the current state of technology, and take advantage of this to eliminate processes susceptible to error or oversight. To this end, a new global SRS calculation and mapping procedure has been developed, and software has been written to implement this process. The approach has been demonstrated on a variety of buildings, and found to give significant advantages by simplifying the overall process and producing higher resolution results.

Acknowledgements

The authors would like to thank Atkins colleagues Nicholas Hucker and Will Jarvis for their assistance in the development of the SRS calculation procedure and software.

References

ASCE 4-16, *Seismic Analysis of Safety-Related Nuclear Structures*, American Society of Civil Engineers (ASCE), 2017

Calvi PM, Sullivan TJ (2014), Estimating floor spectra in multiple degree of freedom systems, *Earthquakes and Structures*, 7: 17-38

Chopra AK (1995), *Dynamics of Structures: Theory and Applications to Earthquake Engineering*, Prentice-Hall

ETC-C:2010, *EPR Technical Code for Civil Works*, AFCEN, 2010

Jiang W, Li B, Xie W-C, Pandey MD (2015), Generate floor response spectra, Part 1: Direct spectra-to-spectra method, *Nuclear Engineering and Design*, 293: 525-546

Jussila V, Li Y, Fülöp L (2016), Statistical analysis of the variation of floor vibrations in nuclear power plants subject to seismic loads, *Nuclear Engineering and Design*, 309: 84-96

Lucchini A, Franchin P, Mollaioli F (2016), Uniform hazard floor response spectra for linear structures, *Earthquake Engineering and Structural Dynamics*, 46: 1121-1140

Lucchini A, Franchin P, Mollaioli F (2017), Spectrum-to-spectrum methods for the generation of elastic floor acceleration spectra, *Procedia Engineering*, 199: 3552–3557

Nigam NC, Jennings PC (1969), Calculation of response spectra from strong-motion earthquake records, *Bulletin of the Seismological Society of America*, 59: 909-922

Sullivan TJ, Calvi PM, Nascimbene R (2013), Towards improved floor spectra estimates for seismic design, *Earthquakes and Structures*, 4: 109-132

Taghavi S, Miranda E (2008), Effect of interaction between primary and secondary systems on floor response spectra, *Proceedings of the 14th World Conference on Earthquake Engineering*, Beijing, China, 12-17 October

Yasui Y, Yoshihara J, Takeda T, Miyamoto A (1993), Direct generation method for floor response spectra, *Proceedings of the 12th International Conference on Structural Mechanics in Reactor Technology (SMiRT 12)*, Stuttgart, Germany, 15–20 August, 367-372