2019 SECED Conference
Earthquake risk and engineering towards a resilient world
9-10 September 2019, Greenwich, London

BIAXIAL STRESS-STRAIN MODEL FOR RUBBER IN SIMPLE SHEAR

Hamid AHMADI[1], Alan MUHR[2] & Enrico TUBALDI[3]

Abstract: *The first part of the paper describes the historical evolution of rubber compounds employed for manufacturing rubber bearings. In the second part of the paper, a comparison is made between the behaviour of rubber compounds exhibiting different levels of damping capacity. For this purpose, a recently proposed model for describing the shear behaviour of rubber, consisting of a parallel arrangement of different rheological elements, is used. It is shown how the parameters of this model vary for the different types of rubber used.*

Introduction

The use, possibly, and the idea, certainly, of Natural Rubber (NR) within structural engineering dates back to Stephenson's Britannia Bridge, taking the railway across the Menai straits. Of the function of the rubber pads, it was written "the vibrations might be cumulative, and increase as the train advanced, until they produce a serious effect….To avoid [this] it was intended to lay the rails on vulcanised India Rubber, about 2 inches thick (Anonymous, 1846). Direct evidence has yet to be found that rubber pads were actually installed, and unfortunately the bridge was destroyed by fire in 1970. Rubber pads may also have been used in two other Stephenson bridges – Conway bridge and the High Level bridge – and were used between pier and deck on the railway viaduct in Melbourne, Australia in 1889 (Anonymous, 1985).

The use of NR-steel laminated bearings by the civil engineering profession goes back to the early 1950s. During the mid-1950s research at the British/Natural/Malaysian Rubber Producers Research Association (BRPRA, NRPRA, MRPRA, being TARRC's former names), led to the development of NR-steel laminated bearings to replace the mechanical bearings then used to support bridge decks. The lamination provided high stiffness vertically to minimize any movement in that direction while maintaining the low shear stiffness of the rubber block, so that the thermal expansion of the bridge deck imposes low loads on the supporting piers.

A collaboration between BRPRA and W.S. Atkins resulted in the design of such bearings for Pelham Bridge in Lincoln UK, manufactured by the Andre Rubber Company and installed in 1956, (Gent, 1959). Suitable formulations for NR were freely published (see Table 1) and promoted by NRPRA. In fact the formulation for the Pelham Bridge rubber (Fuller & Roberts, 1997) resembled a formulation published earlier by NRPRA – as compound C in TBA Information Sheet A appended to Lindley (1962) which was 70IRHD - and would have been similar in properties to EDS38 and EDS16. NRPRA also freely provided design equations, in accord with its mission to promote the industrial use of NR, and seek new technological applications. This was successful, and the use of NR bearings for bridges expanded rapidly, resulting, in less than 10 years from the construction of Pelham Bridge, to 200 bridges supported by NR bearings in the UK alone. Many new producers of such bearings emerged, especially in countries with British influence (Malaysia, New Zealand, Australia, Canada), and often adopted the BRPRA formulation with only minor adjustments.

Such bearings exploit the hyperelasticity (ie elasticity up to very large strains, of the order of 300%) of NR, and in particular its extremely low shear modulus *G*, being in the range 0.3 to 3 MPa. Little filler was used in the formulation, so that the damping was low and the behaviour in simple shear was nearly linear.

A second collaboration between NRPRA, W S Atkins and Andre led to the use of laminated structural bearings, specially designed to give a low vertical stiffness, to isolated buildings from ground-borne vibration generated by underground railways (Waller, 1969). Bridge bearings and vibration isolation bearings usually use relatively lightly filled NR, such as the formulations given

---

[1] Mr, TARRC, Hertford, UK, hahmadi@tarrc.co.uk
[2] Dr, TARRC, Hertford, UK, amuhr@tarrc.co.uk
[3] Dr, University of Strathclyde, Glasgow,UK, enrico.tubaldi@strath.ac.uuk

in Table 1; the rationale on which the formulations is based is explained in NR Technical Information Sheet D133 ((MRPRA, 1984) and additional data on properties are given in the Engineering Data Sheets (MRPRA, 1979-1984).

	EDS36	EDS37	EDS38	EDS32	EDS33	*EDS19*	*EDS16*
ingredient	Parts by weight						
NR (SMRCV60)	100	100	100	100	100	*100*	*100*
ZnO	5	5	5	5	5	*5*	*5*
Stearic acid	2	2	2	2	2	*2*	*2*
N550 carbon black	20	40	60	20	40	*0*	*0*
N330 carbon black	0	0	0	0	0	*0*	*45*
Process oil	2	4	6	2	4	*0*	*4.5*
HPPD	3	3	3	3	3	*3*	*3*
wax	2	2	2	2	2	*2*	*2*
CBS	1.5	1.5	1.5	0.8	0.8	*0.6*	*0.6*
Sulfur	1.5	1.5	1.5	3.25	3.25	*2.5*	*2.5*
IRHD hardness	51	62	70	53	67	*43*	*71*
ASTM4014 grade	0,2,3	0,2,3	0,2,3	5	5	-	-

Table 1 NR formulations promoted by MRPRA for bridge bearings (first 5 materials) and for unspecified engineering purposes (last 2 materials)

In the early seventies, NRPRA embarked on a research project to develop laminated rubber steel bearings to isolate structures from earthquakes. W S Atkins were commissioned and sponsored to advise on the efficacy and requisite properties of such isolators. Their report (Anonymous, 1974) concludes:

- Isolators that are soft horizontally – to protect against the most damaging direction of accelerations – and stiff vertically – to prevent rocking – would be required. These characteristics would be achievable using laminated rubber bearings.
- They should be NR, to minimize creep
- A horizontal natural frequency of about 0.5Hz would be optimum
- The reduction in forces within the superstructure also depends on the damping of the isolators, the higher the better. A value of 10% of critical was thought practicable using known formulation principles, and used for analyses of a five storey hospital-type structure as a test building.
- The forces in the superstructure would be substantially reduced, by a factor of 3 or so, allowing a considerable saving in structural materials which would more than offset the cost of the isolators
- A short basement is necessary at the ground floor, which is an extra cost over directly building on a raft supported by the ground. However, this extra cost would be avoided if the building is already proposed to have a basement or open car parking at ground level.
- The accelerations in the top storeys would be reduced by an even larger factor, around 6 or so, so that isolation offers much reduced forces on the contents of the building; this is a considerable bonus; for example for hospitals, the structural costs are only about 20% of the total cost.

These observations have been borne out remarkably well by subsequent experience, with the exception of the expectation that major savings might be possible on the structural materials required for isolated buildings. The reason for the discrepancy is that code requirements have become more conservative, especially if isolation is used, and the value of protection of contents has as yet been only partially recognized.

In order to achieve the high damping, two very different strategies have evolved: (1) to formulate the NR itself to dissipate energy or (2) to use auxiliary dissipative devices – eg elastoplastic or hydraulic - to provide the requisite damping, in parallel with low damping laminated isolators. TARRC championed the first strategy, which was first used in 1985 for the County Foothills Community Law and Justice (FCLJ) Centre in San Bernadino County in California. The New Zealand Ministry of Works and DSIR championed the second strategy, which was first used for the William Clayton building in Wellington, NZ, 1983, using low damping NR-steel laminated bearings with integral lead plugs for dissipation. This system was commercialized in California by DIS. The success of NR-based isolation systems has been followed by alternative systems, based on sliding isolation bearings, not addressed in this paper.

Types of NR Used for Isolators

Seismic isolation systems based on rubber-steel laminated bearings are of three main types: (1) those using High Damping Rubber bearings (HDR or HDRB) with specially formulated rubber (2) those using Lead-plug Rubber Bearings (LRB or LDRB) in which the rubber has moderate or low damping and a lead plug to enhance damping; (3) those using Low-damping (or linear-elastic) Natural Rubber bearings (LNR) which have almost linear shear stress-strain properties and very little damping, so need to be supplemented with auxiliary devices in parallel – eg hydraulic dampers – if significant damping is required in the design. In all three cases, the laminated bearings support the bulk of the vertical load while accommodating lateral displacement. The steel laminations impart adequate vertical and flexural stiffness to the bearings to ensure that the horizontal compliance is to a good approximation that of the rubber alone in simple shear, and the building load can be borne without risk of lateral instability.

We have explained how low damping bearings developed; that history lent itself to the adoption of similar compounds by all manufacturers. Standards were written to regulate the use of elastomeric bridge bearings, leading towards homogeneity and conservatism across most manufacturers, with a few discrete options for shear modulus *G*. The “high damping rubbers” developed in accord with option (1), however, were regarded with proprietary secrecy by developers, including TARRC and Bridgestone, in accord with traditional practice in the rubber industry, because formulating rubbers is a specialized skill and can lead to a significant commercial advantage if good results are achieved. Development of High Damping (Natural) Rubbers (HD(N)R) also coincided with TARRCs progressive evolution from a government research centre charged with providing free support to would-be users of NR towards an income-generating research facility. Commercial pressures are driving requisite isolator damping levels upwards, not least because higher levels of damping result in reduced horizontal seismic displacements of the isolated structure relative to the ground, which has significant cost advantage. Thus TARRC recently developed “UHDNR” (Ultra High Damping NR) material for seismic isolation bearings, being used by Doshin in the manufacture of isolators for a hospital recently constructed in Jakarta, Indonesia. These isolators have about double the level of damping compared to those used in the FCLJ building in San Bernadino County, Ca. We shall call this proprietary material C25.

It is our purpose in this paper to compare the properties of C25 to those of two low damping NR materials for which formulation details have been published by TARRC (1979); these are given in Table 1 alongside those of materials promoted for use in Bridge bearings. EDS19 may be regarded as at the extreme low-damping low-stiffness end of the low damping material spectrum, and EDS16 at the high end – ie stiffer, higher in damping, and less linear in stress-strain behaviour. The three materials will be compared through the values of the parameters needed to fit a recently published stress-strain model (Tubaldi et al., 2017) to their behaviour in simple shear.

The TRDAM model for rubber stress-strain behaviour in simple shear

The acronym used for this model is taken from the authors of a paper recently published by Tubaldi et al. (2017). The model aims not only to capture elasticity and dissipation, but also the softening that increases with pre-straining to progressively higher strains; it is shown schematically in Figure 1, and expressed in equation 1, providing the shear stress-shear stress constitutive behaviour, τ as a function of γ.

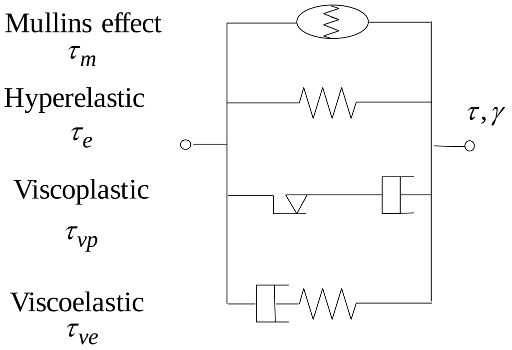


Figure 1. Rheological model for the TRDAM model for HDNR.

$$\tau = \tau_m + \tau_e + \tau_{vp} + \tau_{ve} = (\tau_{me} + \tau_{mvp} + \tau_{mve}) + \tau_e + \tau_{vp} + \tau_{ve} \qquad (1)$$

Hyperelastic component

The hyperelastic (ie non-linear elastic with finite strain capability) component has been fitted with two alternative models, both expressible in terms of strain-energy functions of strain invariant I_1 only, leading to the simple shear stress-strain relationships given in equations 2:

$$\tau_e = c\gamma + b\gamma^3 + a\gamma^5 \qquad \text{Yeoh model} \qquad (2a)$$

$$\tau_e = \gamma[A_{GMS}(\gamma^2 + c^2)^{-\frac{n}{2}} + B_{GMS}|\gamma|^m] \qquad \text{GMS model} \qquad (2b)$$

In Equation (2b), the parameter c_{GMS} is set to a small positive value, to ensure that the modulus at zero strain is finite; the value is not critical, unless there is a need for very accurate modelling at very small strain, and in this work the default value $c_{\text{GMS}} = 0.0005$ was used. .

The two alternative models are designed to allow strain softening at small to moderate strain, and stiffening at high strain. The GMS model (based on the first initials in the authors of Gregory et al. (1997) has been introduced for this paper; having one more parameter, it is somewhat more versatile for fitting, and is also more robust, in that the parameters may be chosen with a free hand, rather than being subject to possibilities such as a negative tangent stiffness for certain combinations.

As explained by Tubaldi et al (2017), the fit for C25 was made on a “fully relaxed” stress-strain graph (their Figure 7), ie, the loading curve incorporates relaxation periods at fixed strain, and similarly the unloading curve incorporates stress-recovery periods at fixed strain. For the other two materials, fits were obtained for the purpose of this paper by comparing the total model to biaxial experimental data.

PARAMETERS FOR FITS TO TRDAM MODEL	**Yeoh[1] hyperelastic parameters; for NeoHooken model, a=0=b; c=G**			**GMS[2] hyperelastic c_{GMS} = 0.0005**			
variable name:	*a*	*b*	*c*	A_{gms}	n_{gms}	B_{gms}	m_{gms}
	MPa	MPa	MPa	MPa	-	MPa	-
C25	0.015	-0.05	0.28	0.24	0.16	0.008	4
EDS16	0.022	-0.055	0.55	0.49	0.1	0.015	4
EDS19	0	0	0.36	0.4	0.1	0.0008[3]	3[3]

Note 1. Yeoh (1990) 2. Gregory et al. (1997). 3. These parameters permit a crude fit to the loading curve, shown in Figure 6, which was subsequently ascribed to viscoplastic parameters

Table 2 Parameters for hyperelastic fits

The GMS model has the advantage that the two individual terms (a softening and a stiffening term) are each thermodynamically plausible. The Yeoh model relies on a negative coefficient *b* for the cubic term to introduce the possibility of softening at intermediate strains, but this relies on the presence of the other two terms, and particular the higher order (quintic) term, to ensure that the stiffness remains positive at all strains. It is therefore not a very robust model, since fitting to a narrow strain range might result in non-plausible extrapolations at higher strain.

Viscoelastic component

Tubaldi et al. (2017) represented the viscoelastic branch using a single Maxwell element; parameters are given in Table 3. If the model is to be used for a wide range of rates or frequencies, however, a four-parameter (but discretised as a Prony series with an infinite sequence of relaxation times) could be substituted instead, following Ahmadi et al.(2008) and Muhr (2011). As EDS 16 and EDS19 have the same cure system and NR matrix, the same viscoelastic model and parameter values may be used for both materials. We currently do not have measurements over a wide enough range of rates and temperatures to fit such a model to C25.

$$\dot{\tau}_{ve} = E_{ve}(\dot{\gamma} - v_{ve}\tau_{ve}) \tag{3}$$

Viscoplastic component

The viscoplastic branch uses a bounding surface plasticity model. This type of model enables the possibility of a plastic stress rate E_p that can rise as the shear strain is increased, in contrast to the overlay of elasto-perfectly plastic models proposed by Ahmadi et al. (2008). A dashpot is added in series with the plastic element, to enable a long time-constant relaxation process to be fitted, such as appears to be necessary (observed for example also by Sternstein and by Muhr (2011), regarding rate dependence of the filler-induced stiffness contribution).

In this paper, a generalisation was made to the viscoplastic model outlined by Tubaldi et al. (2017). In order to better describe the behaviour of EDS19 at very large strains (up to double those to which Tubaldi et al. (2017) subjected C25 and applied their model), a quartic term in plastic simple shear strain has been added to the expression for the bounding surface, it previously being parabolic:

$$R = \xi_0 + \xi_1\gamma_p^2 + \xi_2\gamma_p^4 \tag{4}$$

$$\dot{\tau}_{vp} = \left(\frac{dR}{d\gamma_p} + \left(h_0 + h_1|\gamma_p|\right)\mathrm{sign}(\dot{\gamma}_p)\,.\left|\tau_p - R.sign(|\dot{\gamma}_p|)\right| \right).\ \dot{\gamma}_p \tag{5}$$

Furthermore, ξ_0 was set to zero for EDS19, in accord with the observation that the material is nearly elastic up to moderately large strains. This was not without care, because a consequence is that non-physical retraction behaviour occurs (energy generation on retraction) unless a criterion is satisfied regarding the h_0 and h_1 values.

PARAMETERS	bounding surface			plastic modulus		Inverse plastic viscosity	Viscoelastic	
	ξ_0	ξ_1	ξ_2	h_0	h_1	v_1	E_2	v_2
	MPa	MPa	MPa			$MPa^{-1}\cdot s^{-1}$	MPa	$MPa^{-1}\cdot s^{-1}$
C25	0.14	0.08	0	3.5	1.5	0.4	0.07	8.5
EDS16	0.06	0.03	0	3.5	1.5	0.0865	0.07	10.5
EDS19	0	0	0.0011	0.1	0.4	0.004[1]	0.01	10.5

Note 1. Needed only for fit in Figure 7, where strain rate was ~ 1/30th of that in all biaxial plots

Table 3 Values of viscoelastic and viscoplastic parameters to fit TRDAM model to experiment

Mullins stress softening

In Figure 1 the Mullins effect is schematically depicted by a fragile structure - an eggshell - which may initially bear considerable stress but is prone to progressively fracture, resulting in a decline towards zero of the extra stress bearing capacity that it imparts. Tubaldi et al. (2017) used a damage approach, scaled to the post-softened behaviour outlined above, to describe the evolution of extra stresses that are observed when the material is initially strained. These extra stresses have the same three parallel branches as outlined above for the equilibrium elements, but fade asymptotically to zero as the maximum strain is increased to the bound γ_{mod} envisaged for applicability of the model, and as the number of exposures to high strain increases:

$$\tau_m = a_e(1-q_e^{\pm})\tau_e + a_{vp}(1-q_{vp})\tau_{vp} + a_{ve}(1-q_{ve})\tau_{ve} \tag{6a}$$

where the generic damage parameter $q_\bullet$ evolves according to

$$\dot{q}_\bullet = \zeta_\bullet\left(\left(\frac{\gamma}{\gamma_{\text{mod}}}\right)^{\beta_\bullet} - q_\bullet\right).|\dot{\gamma}| \ \text{ if } \ q_\bullet < \left(\frac{\gamma}{\gamma_{\text{mod}}}\right)^{\beta_\bullet} \ \text{ else } \ \dot{q}_\bullet = 0 \tag{6b}$$

Thus after scragging sufficiently in all possible directions to a maximum strain γ_{max}, equation (6a) will become

$$\tau_m = \left(1-\left(\frac{\gamma_{\text{max}}}{\gamma_{\text{mod}}}\right)^{\beta}\right)\left[a_e\tau_e + a_{vp}\tau_{vp} + a_{ve}\tau_{ve}\right] \tag{6c}$$

In equation (6b) γ_{mod} is the maximum strain for which the model is valid; softening depends on the strain history, and asymptotes to a limit state. The experimental data does not justify distinction between β_{e} , β_{vp} and β_{ve} , between ζ_{vp} and ζ_{ve} , or between a_{vp} and a_{ve} . However, the data do show that in the case of τ_{e} the softening is NOT isotropic as for the dissipative stress contributions: no softening of the elastic stress for negative strain is caused by a strain history consisting entirely of positive strain, and vice versa.

Tubaldi et al. (2017) explain how equation (6b) can be modified to describe such generation of anisotropy by softening. The form of equations (6) implies that if softening is run to completion at a given maximum strain γ_{max} (applied in all possible directions), then the behaviour is simply as given in equations (2) to (6) but with the factors (1-$q_\cdot$) replaced by a pre-factor of (1-($\gamma_{\text{max}}/\gamma_{\text{mod}})^{\beta}$). Above $\gamma = \gamma_{\text{max}}$ softening will resume. Note that the assumption is made that the scragging below γ_{max} will soften the material for $\gamma > \gamma_{\text{max}}$, so that the continued loading curve will be continuous with the loading curve for the material fully scragged to γ_{max} , but there will be no rise towards the stress corresponding to γ_{max} attained during primary loading. This is in contrast to some models for the Mullins effect (eg Ogden & Roxburgh, 1999), and resolving this point calls for further experimental data, and possible adjustment to the model if needed.

Table 5 gives the values of the parameters to fit the model to the experimental data. As EDS19 showed no stress-softening at the strains used in the experiments, the parameters a_{e} and a_{v} for it are zero and it is not included in the Table.

PARAMETERS FOR FITS TO TRDAM MODEL	a_e	ζ_e	β	a_v	ζ_v
C25	1.7	0.25	0.4	2.2	0.13
EDS16	0.35	0.1	0.5	0.595	0.1
EDS19	-	-	-	-	-

Table 5 Values of stress-softening parameters to fit model to experiment

Comparison of the TRDAM model to simple shear test data

Ragni et al. (2018) showed that the TRDAM model could be extended to biaxial simple shear by adding contributions to shear stress from all directions in the plane of shear. Tests done at TARRC were performed at 0.5Hz in biaxial simple shear using the rig described by Ahmadi et al., (2019).

To indicate the fit achieved with the model, square and figure of 8 trajectories with a maximum strain of 100% were applied as shown in Figure 2. The results are given in Figures 3 to 5. Figure 3, for C25, reveals very striking non-linearity and biaxial coupling effects.

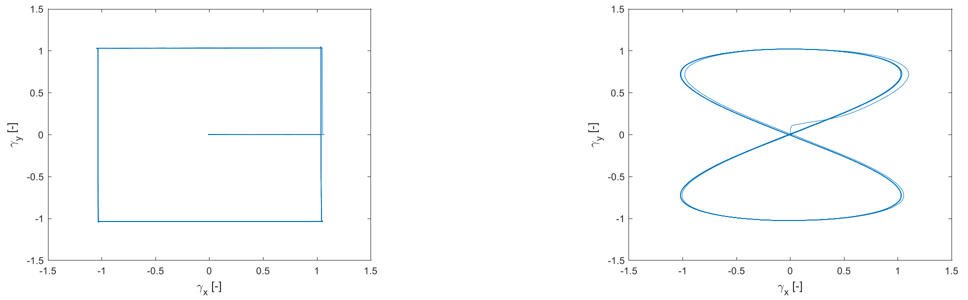


Figure 2 Left, square trajectory; right, figure of 8 trajectory.

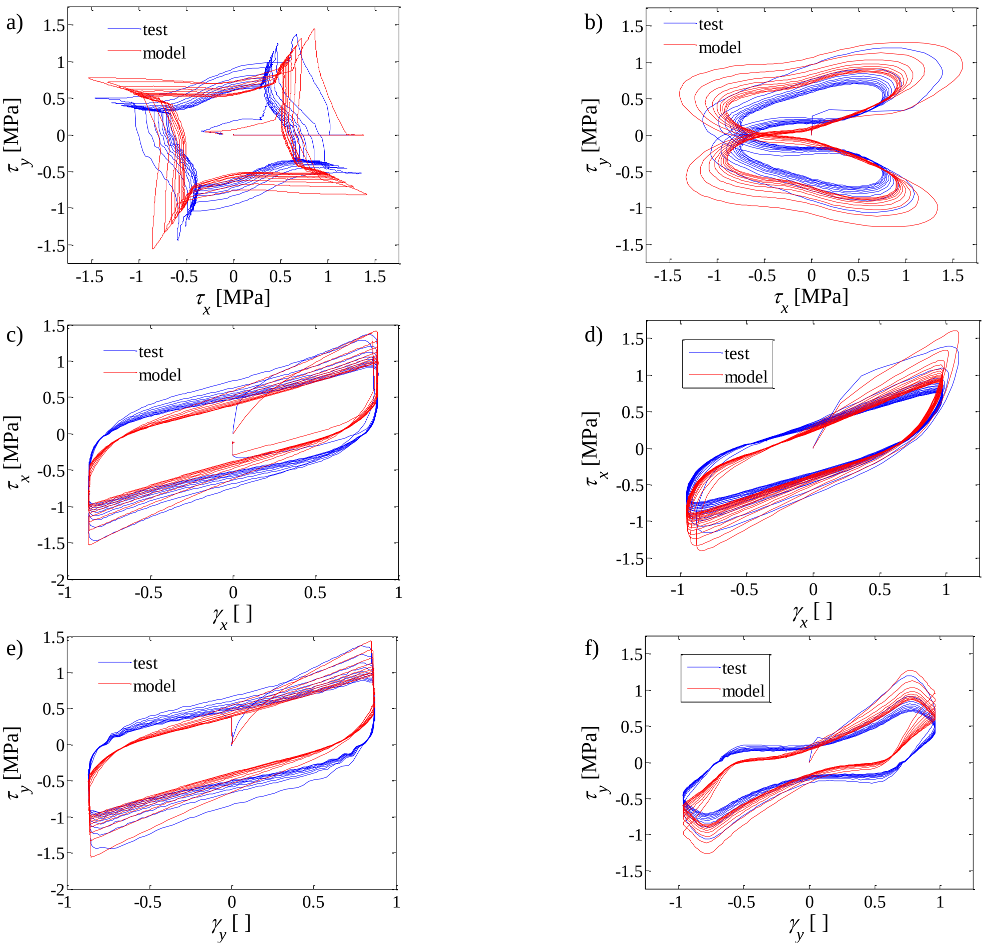


Figure 3 C25 comparison of test and model; biaxial simple shear trajectories with 100% maximum strain, left, square; right, figure of 8

For EDS16 (Figure 4), the nonlinearity and coupling effects are much less than for C25, but there is still a significant level of energy dissipation. It is expected that the materials in Table 1 of similar hardness (70 IRHD) to EDS16 will have similar hysteresis and non-linearity in stress-strain behaviour.

The test data on EDS19 show (see Figure 5) that up to moderately large strains its behaviour is close to isotropically elastic. However, Figure 6 shows data for EDS19 in uniaxial simple shear up to 500% revealing that this is not the case at very large strains. It is apparent after all that unfilled NR, although elastic with nearly linear (neo-Hookean) stress-strain behaviour in simple shear up to moderate strains, shows at higher strains (a) upturn in stiffness (b) hysteresis. The underlying hyperelastic behaviour of EDS19 appears to be slightly softening relative to neo-Hookean (see blue solid line in Figure 6). A stiffening term (eg he second term in the GMS model) can be added to fit the upturn in stiffness at high strain (see green dashed line). If instead it is subtracted, it gives a lower curve which is somewhat suggestive of the retraction curve of the real material (see red dashed line), suggestion that both the stiffening and the hysteresis have the same underlying cause, that is not elastic. The cause may be identified with non-equilibrium strain crystallisation. The TRDAM model can give a good approximation to such behaviour, as shown in Figure 7; the green and red solid lines in Figure 6 give respectively the loading and unloading boundary surfaces for the TRDAM model proposed for EDS19.

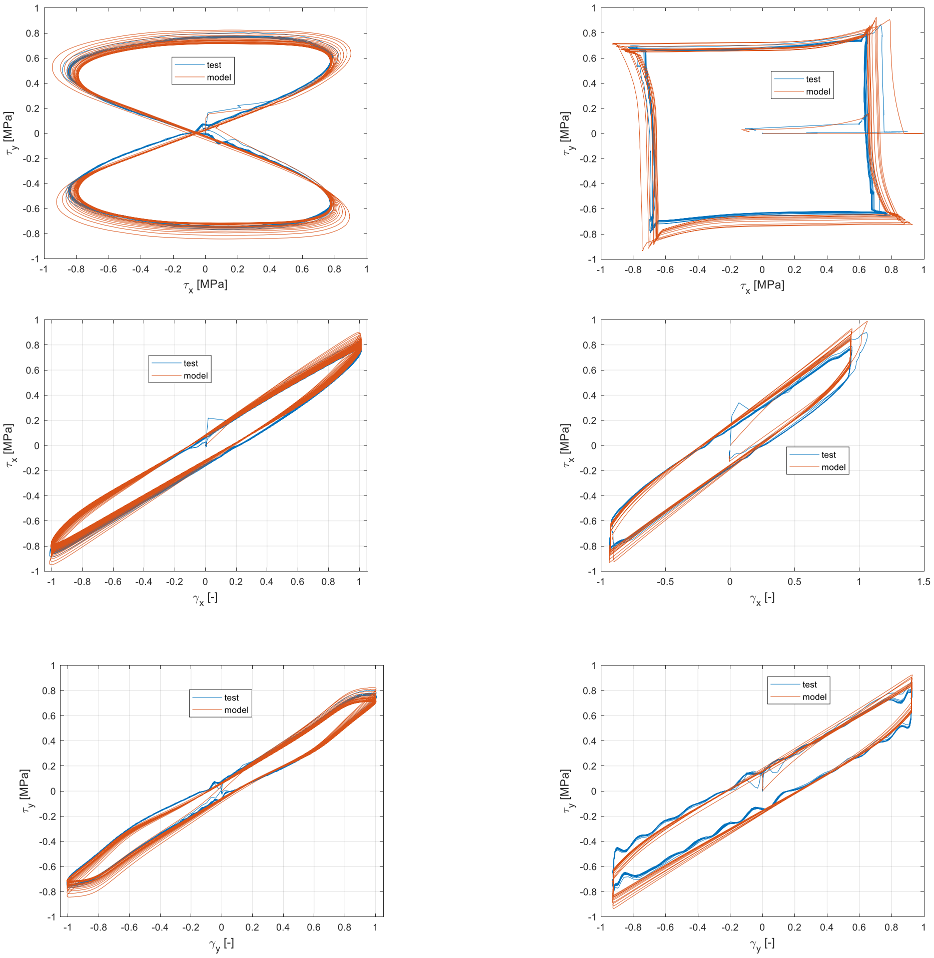
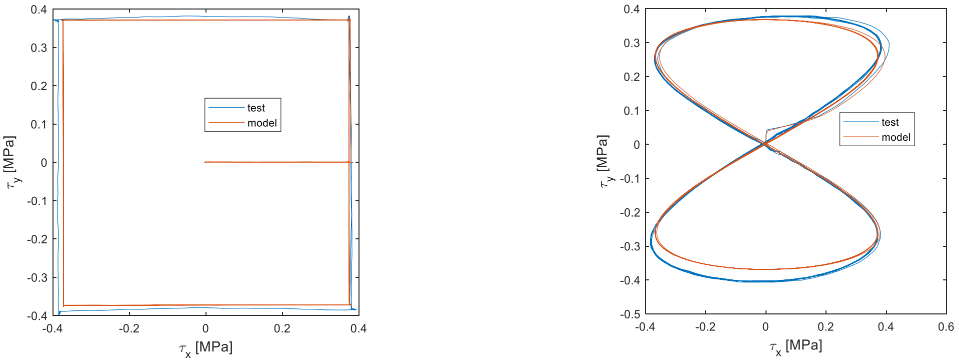


Figure 4 EDS 16 biaxial simple shear trajectories left, square; right, figure of 8



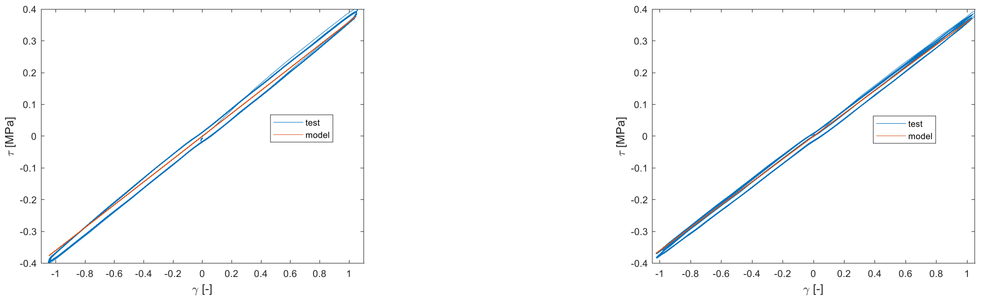


Figure 5 comparison of test and model for EDS19, subjected to the figure of eight strain trajectory The plot of τ_x versus γ_x looks near-identical

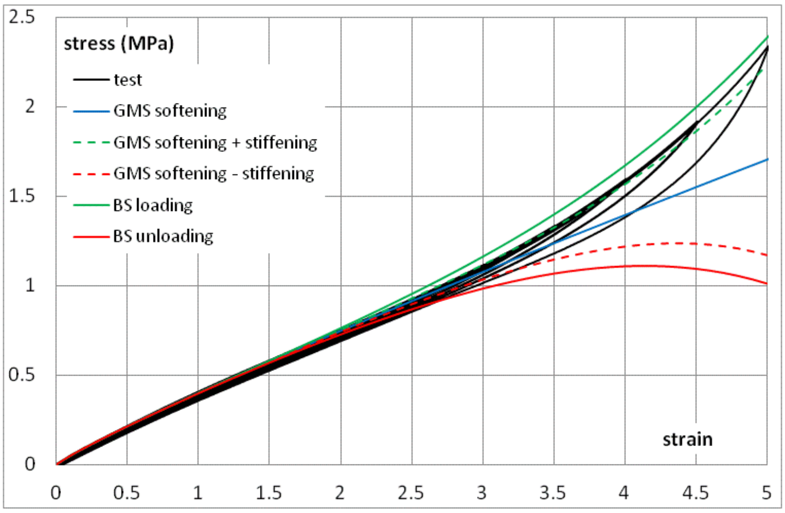


Figure 6 Attempts at hyperelastic fit over an extended strain range to EDS19 subjected to simple shear (superimposition of 3 cycles to each of shear strain maxima in steps of 50% up to 500%. Strain rate $0.1s^{-1}$.

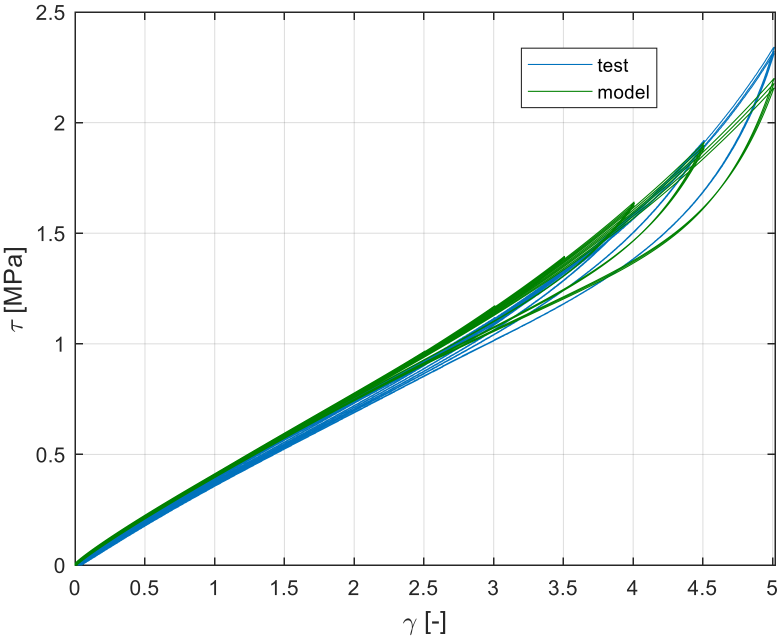


Figure 7 Application of TRDAM model to fit over an extended strain range to EDS19 subjected to simple shear (strain rate $0.1s^{-1}$)

Summarising Discussion

With suitable minor modification and choice of appropriate parameters, the TRDAM model may be used to describe the properties of HDNR and “linear” NR formulations at all strains likely to be of interest in engineering applications. It is noted that at sufficiently high strains (>300% in simple shear) even low damping NR exhibits strain stiffening and hysteresis. It is possible that this phenomenon can be tailored by formulation of the NR, and harnessed to the benefit of isolation systems, that provide extra stiffness and hysteresis at very large displacements that may occur in rare seismic events above the design level (Kelly, 2007). It is acknowledged that the treatment of rate effects in the model, particularly for unfilled NR at high strain, may require further elaboration to cover tests over a broad range of rates. For this reason, it may not be reliable to compare the value of v_1 in Table 3 for EDS19 to those given for the other materials.

The authors wish to thank Luigi Zappa for the carrying out the tests reported in Figure 7.

References

Anonymous (1846) *Builder*, **4**, 481 Anonymous (of W S Atkins Research & Development), (1974), Preliminary report on the use of rubber springs to protect buildings in earthquake zones, *Report to MRPRA*

Anonymous, (1985) Rubber bridge pads from the Victorian age, *Rubber Developments*, **38**, 36-38

Ahmadi H R, Kingston J G R & Muhr A H, (2008) Dynamic properties of filled rubber I Simple model, experimental data and simulated results. *Rubber Sci & Tech*, **81**, 1 - 18

Ahmadi H R, Muhr A H, Orlando V, Poisson J-L P & Stephens I J (2019) Biaxial testing facilities for anti-seismic devices

Derham C J, Thomas A G & Kelly J M, (1978) The design and use of rubber bearings for vibration isolation and seismic protection of structures, *Eng. Struct*, **2**, 171 - 175

Derham C J, Thomas A G, Eidinger J M & Kelly J M, (1980) NR foundation bearings for earthquake protection – Experimental results, *Rubber Chem & Tech*, **53**, 186 – 209

Gent A N (1959) Rubber bearing for bridges: design considerations. *Rubber Journal*, October, **137**, 420 - 422

Gregory I H, Muhr A H & Stephens I J (1997) Engineering applications of rubber in simple extension, *Plastics, Rubber & Composites*, **27**, 118 - 122

Kelly JM (2007) email communication to A H Muhr

Lindley P B (1962) NR in Bridge Bearings, *Technical Bulletin no. 7,* publ. NRPRA

MRPRA, (1984) Bridge bearings – formulations meeting material requirements of ASTM D4014 and BS5400:9.2 *NR Technical Information Sheet, Dry Rubber Series,* D133, publ. MRPRA

Muhr A H (2009) Fitting a viscoplastic time-domain model to equivalent viscoelastic materials data, ECCMR 2009

Ogden R W & Roxburgh D G (1989) A pseudo-elastic model for the Mullins effect in filled rubber. *Proc Roy Soc Lond* **A455**, 2861-2877

Ragni L, Tubaldi E, Dall'Asta A, Ahmadi H R, & Muhr A H (2018), Biaxial shear behaviour of HDNR with Mullins effect and deformation-induced anisotropy, *Engineering Structures*, **154**, 72 – 84

TARRC 1979-1986 Engineering Data Sheets, *publ TARRC/MRPRA*

Tubaldi E, Ragni L, Dall'Asta A, Ahmadi H R, & Muhr A H (2017) Stress-softening behaviour of HDNR bearings: modelling and influence on the seismic response of isolated structures, *Earthquake Engineering & Structural Dynamics,* **46**, 2033-2054

Waller, (1969) Building on Springs, Pergamon, Oxford, UK.

Yeoh O H (1990) Characterisation of elastic properties of carbon-filled rubber vulcanizates, *Rubber Chem & Tech,* **63**, 792 - 805