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FOOTFALL INDUCED VIBRATIONS ON STEEL-CONCRETE COMPOSITE FLOORS: SERVICEABILITY PERFORMANCE UNDER WALKING LOAD UNCERTAINTY

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Abstract: *In view that human-induced vibrations could impose the feeling of discomfort to users of lightweight floor systems, a probabilistic serviceability performance assessment is carried out, considering a set of conventional single-unit steel-concrete composite floors that conform to the AISC design guidelines. To investigate the parameters that are likely to undermine the vibration performance of floors due to a single human walking, a structural model is formed to numerically extract key engineering floor demand parameters, such as the vertical peak floor acceleration and the vertical peak floor displacement. The effect of various footfall load modelling options is examined by generating acceleration response spectra against a range of realistic step frequencies. The floor response obtained via the structural model for a range of walking frequencies is then approximated via a polynomial fit. Exploiting suitable step frequency distributions that are available in the literature, walking frequency samples are generated and the floor acceleration responses are computed on the basis of the polynomial fit. This process allows to estimate, for a considered walking load model, the distribution of the floor accelerations should the randomness in the step frequency of its users is considered. The results reveal that, under certain walking load modelling assumptions, the probability of an AISC-conforming floor exceeding the suggested comfort acceleration limit of 0.5%g could be rather significant.*

Introduction

Composite steel-concrete floor systems are widely used in construction for achieving long-span floors with a low number of intermediate columns. The design of such slender and lightweight floor systems is typically governed by serviceability limit state requirements, associated with deformations, human comfort perception, and vibration tolerances. To guide designers through the process of delivering floors that are not prone to vibrations imposing a feeling of discomfort, several design guidelines of variable complexity have been developed in the past few decades (e.g., AISC, 2016; Smith et al., 2009). In their simplest form, such guidelines adopt several deterministic assumptions regarding floor damping, imposed loads, connection rigidity under service loads, step frequency, footpath and human weight.

Further to the above, composite steel-concrete floors are also characterised by low self-weights and damping ratios compared to the ordinary reinforced concrete ones. Hence, in view of these distinct properties, contemporary composite steel-concrete floors are more prone to humaninduced vibrations that could cause discomfort to their users (i.e., vibration serviceability issues). Discomfort, as well as the perception of annoying vibrations in general, is a rather complex and subjective matter. For instance, tolerance to vibrations is affected by the type of the environment, with the acceptable limits being higher for more active places (e.g., shopping malls) and lower for less active ones (e.g., hospitals, offices). This condition is reflected in the acceleration limits that are defined in AISC/CISC Design Guide 11 (AISC, 2016) and are herein presented in Figure 1.

In general, based on the magnitude of their fundamental frequency, floors are characterised as either low- or high-frequency ones. Although the exact frequency threshold for characterising a floor as low- or high-frequency varies in the literature, floors with first mode natural frequencies in excess of 10Hz are typically characterised as high-frequency ones (Brownjohn and Middleton,

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2008). The implications of such a classification are not restricted to the realm of theory, but are rather directly reflected to the anticipated vibration response under human-induced excitations. In particular, low-frequency floors are prone to resonant build-up, a condition that occurs when the step frequency or a multiple of the latter (i.e., harmonic) matches the eigenfrequency of a floor mode and especially that of the floor's natural frequency. By contrast, high-frequency floors are not prone to resonant build-up since no frequency matching can practically occur, and thus, they exhibit an impulsive response.

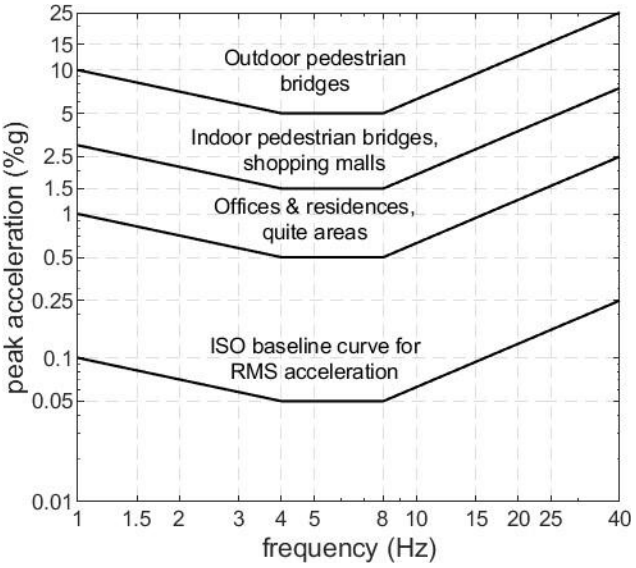

Figure 1: Recommended acceleration tolerance limits for human comfort (after AISC, 2016)

Sources of uncertainty

The vibration performance of steel-concrete composite floors under human walking is affected by a variety of factors, such as the dynamic properties of the floor, its damping ratio, the weight of the individual, the step frequency and the step length, among others (e.g., walking path, load model). One of the major causes of annoying vibrations due to human activity refers to the case where the beat of the activity of one of its harmonics matches the modal frequencies of the floor.

The load imposed on a floor due to a human walking, comprises three components in the associated lateral, longitudinal and vertical direction. However, the first two components are disregarded in this study, and only the most important (at least for the case of floors) vertical one was considered. This study is also restricted in the realm of a single person excitation, which is deemed to be the standard design scenario in the case of office floors (Pavic and Reynolds, 2002). Yet, even for this simple case, the actual step frequency, step length and human weight are random variables in a population of different individuals. For instance, by means of walking load experiments on 61 test subjects and 2204 records, Chen et al. (2014) found that the walking step frequency f_s of normal walk approximately follows a normal distribution, with a mean value of 1.937Hz and a standard deviation of 0.296Hz. The mean of the aforementioned distribution complies well with the 2.0Hz pacing rate reported before by Bachmann and Ammann (1987) for normal walking conditions. Other researchers, such as for instance Matsumoto et al. (1978), proposed similar normal distributions for normal walking (pacing rate 2.0Hz and a standard deviation of 0.18Hz). Similarly, on account of measurements conducted in an office building at Delft, the walking frequency was approximated with a lognormal distribution having a mean of 2.0Hz and a CoV of 8.5% (Smith et al., 2009). The SCI Publication P354 (Smith et al., 2009) states that although the pace frequencies of walking activities may range from 1.5Hz to 2.5Hz the most probable range is between 1.8Hz to 2.2Hz.

Damping is another factor that plays a determinant role in the vibration assessment of floors, as it defines to a large extent the magnitude of the response in low frequency floors. In high frequency floors damping was found not to affect the initial peak response due to the footfall impact (Murray et al., 2018); yet, in both cases it affects the decay of the motion. Apart from the material type, damping varies substantially between floors having connections with different rigidity, partition walls, equipment or furnishing, suspended ceilings as well as stationary humans (Chen et al., 2018). In general, compared to ordinary concrete floors, steel-concrete composite ones are characterised by lower damping levels, that consequently could lead to more severe vibrations and hence discomfort to their users. Hewitt and Murray (2004) also indicated the lack of paperwork in modern offices as a reason for the lower damping levels. The methodology that is presented by Feldmann et al. (2009) for the design of floors against human-induced vibrations defines the system damping as the sum of the contribution of three individual factors, that are the structural damping which varies for different construction materials, the damping due to furniture as well as the damping due to finishes. In fact, the importance of damping in the floor vibration response is also further highlighted by the fact that the increase of damping is among the most important retrofit measures against annoying human-induced floor vibrations. This can be attained by, e.g., changing the position of the non-structural elements or through utilising tuned mass dampers (Smith et al., 2009).

This study proposes a methodology to conduct a probabilistic floor vibration assessment. Although the source of uncertainty is currently restricted in the step frequency variability, the proposed methodology can incorporate other random variables (e.g., damping, human weight, etc.) to eventually shed light in the following issues: (a) the extent to which different uncertainty sources are likely to undermine the vibration performance of a steel-concrete composite floor conforming with recent design guidelines; (b) estimate the variation of the peak floor acceleration under uncertainty as well as the probability of exceeding the acceptable acceleration limits for human comfort; and (c) provide insights on the composite floor properties that could be modified to enhance the floor vibration performance. The focus is on low-frequency floors that are prone to resonance phenomena, in particular composite steel-concrete floor systems that are commonly used in modern construction, yet they often have relatively low natural frequencies that lie within the frequency range likely to be affected by human activities (e.g., walking, running, dancing).

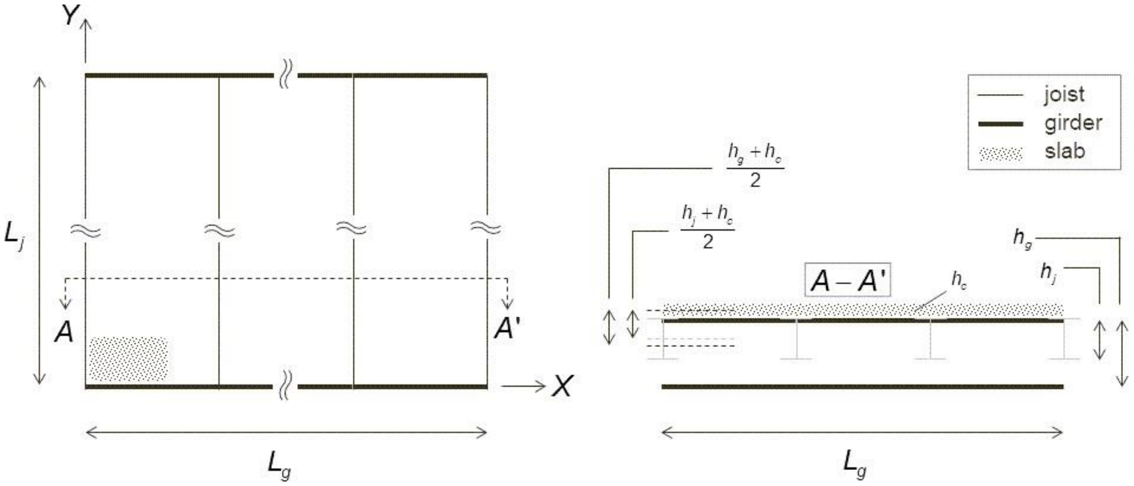

Figure 2: Generic drawing of an $L_g \times L_j$ steel-concrete composite slab; (left) plan view; (right) AA' cut view

Case study

Two single-unit steel-concrete composite floors are examined herein. Floor A (Mello et al., 2008) is used to present the procedure that is followed to assess from a probabilistic standpoint the level of discomfort due to human-induced walking vibrations, while Floor B (Chen et al., 2018) is used to offer additional analysis results. A generic drawing of the floors considered, is illustrated in Figure 2. It comprises two steel girders having a span of L_g, four girders with a span of L_j and a $h_c =$ 150mm thick concrete slab. All cross-section properties for both beams and columns are summarised in Table 1.Floor A is borderline acceptable according to the AISC (2016) design guide (Mello et al., 2008), while Floor B comprises a more robust system (Chen et al., 2018).

	Member	**Cross section**	**Height (mm)**	**Flange width (mm)**	**Flange thickness (mm)**	**Web thickness (mm)**	**Length (m)**
Floor A	Girders	VS I 550□64	550	250	9.5	6.3	9.0
	Joists	VS I 450□51	450	200	9.5	6.3	6.5
	Columns	CS I 300□62	300	300	9.5	8.0	5.0
Floor B	Girders	I-45	450	160	14.2	8.6	7.0
	Joists	I-40	400	155	13.0	8.0	8.2

	Columns	I-40	400	155	13.0	8.0	3.0

Table 1: Member properties of the case study floors (Chen et al., 2018; Mello et al., 2008)

Step force modelling

Several different options are available in the literature regarding the modelling of the step forces. One of the earliest models assumes the step force as perfectly periodic, thus allowing the respective loading function *F*(*t*) to be expressed through the following Fourier series (e.g., Bachmann and Ammann, 1987):

$$F(t) = W\left[1 + \sum_{i=1}^{n} \alpha_i sin(2\pi i f_s t + \varphi_i)\right] \tag{1}$$

In Equation (1), *W* is the weight of the individual (often assumed between 700N and 800N), i is the harmonic component, t is the time in seconds, f_s is the step frequency in Hz, α_i is the dynamic coefficient of the i^{th} harmonic and φ_i is the phase angle of the i^{th} harmonic.

An alternative walking load model is proposed by Feldmann et al. (2009). In this model, the load of a person walking on a floor, is approximated by a series of steps, with the contact force of each step estimated via the following formula:

$$F(t) = W \sum_{i=1}^{8} K_i t_i \tag{2}$$

The coefficients K_i that are used to evaluate Equation (2) are summarised in Table 2.

	$f_s \leq$ **1.75Hz**	**1.75Hz** $< f_s <$ **1.75Hz**	$f_s \geq$ **2.00Hz**
K_1	$-8 \cdot f_s + 38$	$24 \cdot f_s - 18$	$75 \cdot f_s - 120.4$
K_2	$376 \cdot f_s - 844$	$-404 \cdot f_s + 521$	$-1720 \cdot f_s + 3153$
K_3	$-2804 \cdot f_s + 6025$	$4224 \cdot f_s - 6274$	$17055 \cdot f_s - 31936$
K_4	$6308 \cdot f_s - 16573$	$-29144 \cdot f_s + 45468$	$-94265 \cdot f_s + 175710$
K_5	$1732 \cdot f_s + 13619$	$109976 \cdot f_s - 175808$	$298940 \cdot f_s - 553736$
K_6	$-24648 \cdot f_s + 16045$	$-217424 \cdot f_s + 353403$	$-529390 \cdot f_s + 977335$
K_7	$31836 \cdot f_s - 33614$	$212776 \cdot f_s - 350259$	$481665 \cdot f_s - 888037$
K_8	$-12948 \cdot f_s + 15532$	$-81572 \cdot f_s + 135624$	$-174265 \cdot f_s + 321008$

Table 2. K_i coefficients for the Feldmann et al., (2010, 2009) load model

In this study, to undertake the numerical analyses for determining the response of the investigated composite floor, the load model proposed by Feldmann et al. (2009) is adopted and the load duration of a single footfall (t_s) is computed as:

$$t_s = 2.6606 - 1.757 \cdot f_s + 0.3844 \cdot f_s^2 \tag{3}$$

The length of each step (L_s) can be estimated as (Sedlacek et al., 2006):

$$L_s = \frac{v_s}{f_s} \tag{4}$$

In Equation (4) v_s is the velocity of the individual walking on the floor, which can be evaluated according to the following relationship (Bedon, 2022):

$$v_s = 1.67 \cdot f_s^2 - 4.83 \cdot f_s + 4.5 \quad (5)$$

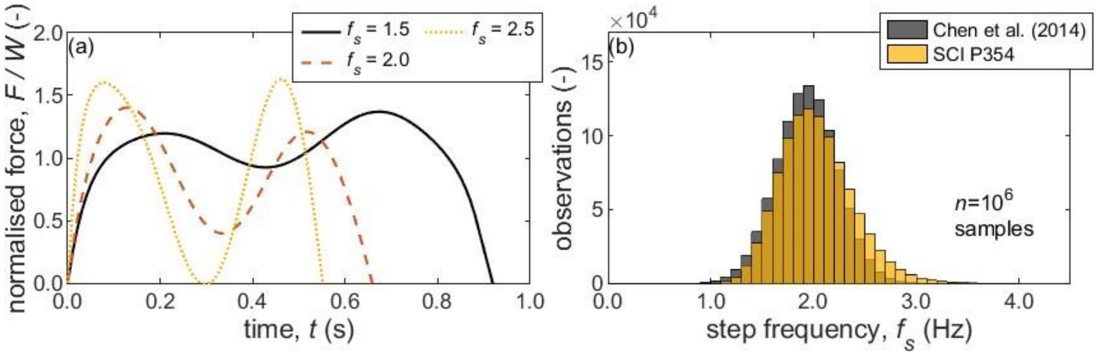

Figure 3: (a) Single-step load functions for step frequencies of 1.5Hz, 2.0Hz and 2.5Hz (Feldmann et al., 2009)*; (b) step frequency distributions (Chen et al., 2014; Smith et al., 2009)*

Figure 3(a) presents the normalised load timeseries functions for a single footstep and three indicative step frequencies. It is provided side by side with two step frequency histograms that are generated according to distributions of the step frequencies that are available in the literature by Chen et al. (2014) and Smith et al. (2009).

Modelling

The numerical investigation of the case study steel-concrete composite floors is carried out using the OpenSees software platform (McKenna, 1997). In the adopted computational model, girders (i.e., primary steel beams), joists (i.e., secondary steel beams) and steel columns are modelled with elastic beam-column elements that are readily available in the OpenSees element library. The composite slab is modelled by means of a grillage of interconnected elastic beams. Each grillage node is assigned a mass that is calculated based on the respective tributary area. As the girders, joists and grillage beam elements have their centroids at different elevations (Figure 2), vertical rigid links are used to connect the nodes of the concrete slab with those nodes of either the girders or the joists that are in the same position but at a different elevation (Figure 4). Both girders and joists are discretised following the mesh size that was finally adopted for the slab grillage, in view of the outcomes of a sensitivity study, that are presented later on in this manuscript. A Rayleigh damping approach is adopted, assigning a damping ratio of 3% in the first and second vibration modes.

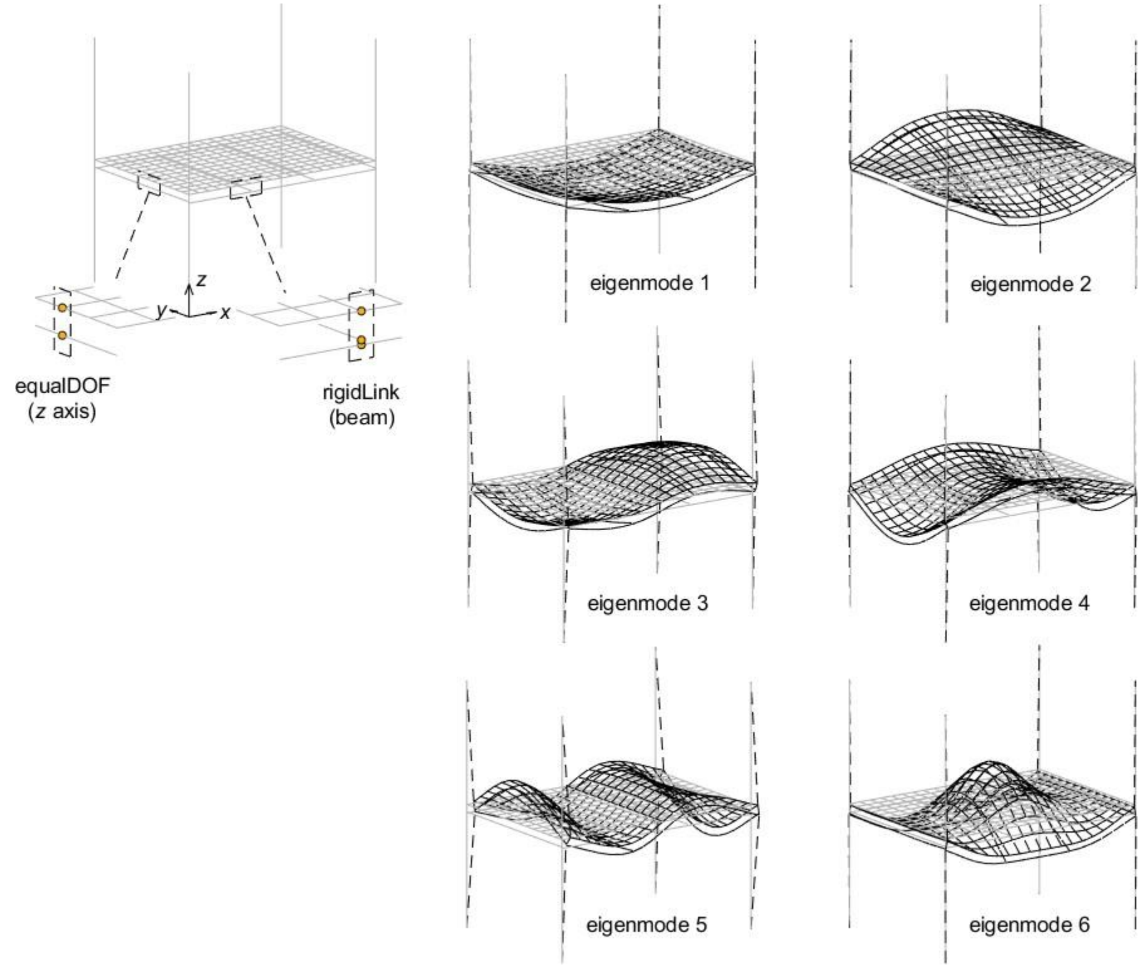

Figure 4: 3D floor model and mode shapes 1-6 of the Floor A case study; figure shows the mode shapes of the 0.5m□0.5m mesh density model

Modal analysis

To determine the dynamic properties of the case study steel-concrete composite floors (natural frequencies and mode shapes) modal analysis is performed. The mode shapes of the case study Floor A are presented in Figure 4. A parametric analysis is undertaken for this floor configuration to determine the optimum refinement for the rectangular mesh that is used for modelling the slab. Using a 1:1 element aspect ratio for the grillage (and thus girders and joists), the mesh size sensitivity study employs models with element sizes varying from 1.0m□1.0m to 0.04m□0.04m. According to the results shown in Figure 5, a mesh size of 0.1m□0.1m is fine enough to yield robust estimates for the modal frequencies of the investigated floor, in the sense that further refinement does not result in any notable difference in the frequency estimates. Hence, a grillage size of 0.1m□0.1m is adopted, which also serves well the requirements that stem from the need to apply the footfalls across a walking path at certain distances (Bedon, 2022; Cai et al., 2020). The frequencies of the first six modes for the investigated floors are summarised in Table 3.

Floor / Mode	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**
Floor A	7.86	14.99	15.95	22.51	31.96	33.78
Floor B	7.90	12.60	19.49	19.74	32.74	33.32

Table 3: Eigen-frequencies 1-6 of the floor case studies

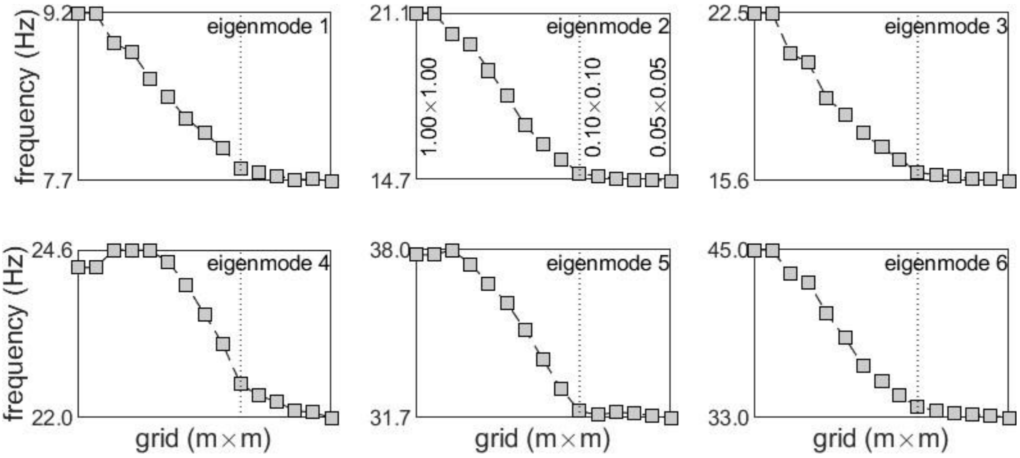

Figure 5: Mesh density sensitivity for the first six eigenmodes of the Floor A case study

Response history analysis

To evaluate the dynamic response of the case study floors, response history analysis is employed utilising the Newmark time integration algorithm. To properly simulate each footfall on the composite slab, the force timeseries are evaluated using Equation (2) and are then applied on the grillage nodes of the 3D floor model shown in Figure 4. For a certain footpath (e.g., along the X axis of the floor), a lateral distance of footfalls (D_s) is considered, using a value of 0.2m (Bedon, 2022). Moreover, the overlap t_0 between two consecutive footsteps (Sedlacek et al., 2006) is also taken into account as:

$$t_0 = t_s - \frac{1}{f_s} \tag{6}$$

An illustrative description of the aforementioned procedure is offered through Figure 6, where consecutive force functions are presented versus time in Figure 6(a) and the nodes of the entire footpath on the 3D floor model, where force functions are applied, in Figure 6(b). The integration time of the transient analysis is 10s, which includes the duration of the footsteps plus a few extra seconds for free vibration. Indicative response histories for floor acceleration and displacement are also provided in Figure 6(c, d) at the locations where maximum response is recorded.

Load model effect

The example presented in Figure 6 features the case where the step force is explicitly modelled during the entire footpath, accounting for both lateral distance and time overlap between two consecutive footsteps. As other, less complex, techniques are often utilised by many researchers (Bedon, 2022; Cai et al., 2020; Mello et al., 2008), the impact of other footfall load modelling assumptions on the evaluated floor response is also examined. Using the load function of Equation (2), the effect of the time overlap between two consecutive footsteps, as well as that of the lateral distance between left and right footsteps, are investigated. The structural model outlined in a previous section of this paper is analysed (using the 0.1m□0.1m mesh density) for a range of plausible walking frequencies (i.e., 1.5-2.5Hz) in view of generating a spectrum of floor responses. Figure 7 presents the results of this investigation, featuring the peak floor acceleration

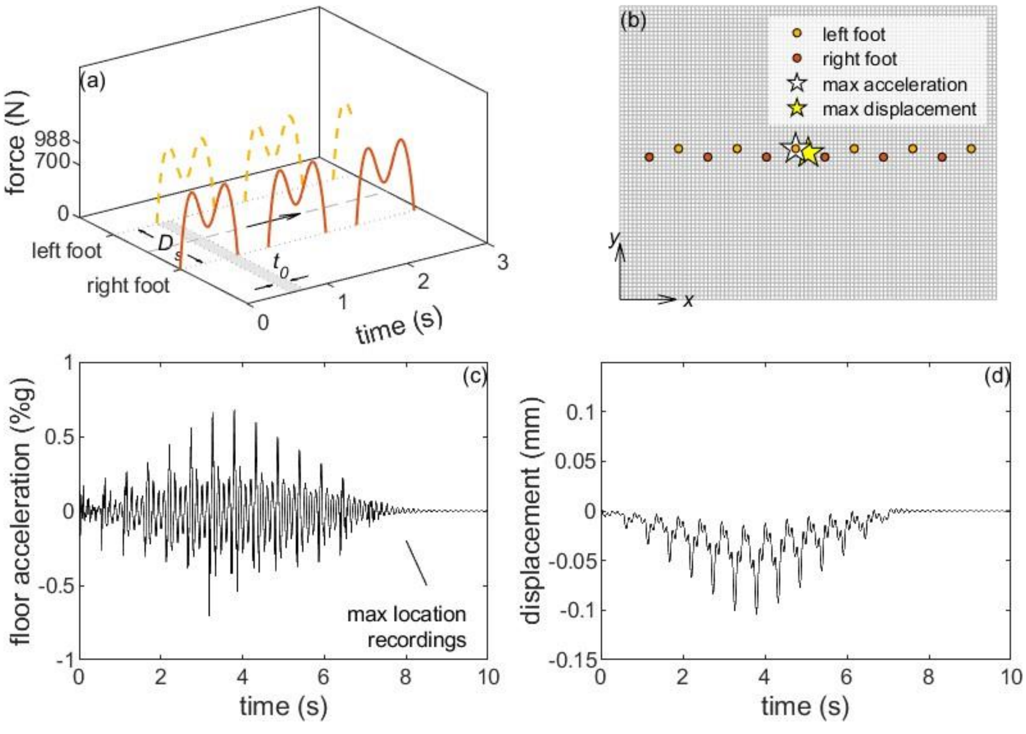

Figure 6: (a) Loading input; (b) plan view of the 3D Floor A model featuring the footpath along the x-axis and the maximum acceleration and displacement locations; (c) maximum acceleration time history at the location of the maximum response; (d) displacement time history at the location of the maximum response

and the root mean square (RMS) floor acceleration as suitable engineering demand parameters. The latter is estimated as:

$$RMSa = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a^2(t)\, dt} \tag{7}$$

In Equation (7), $a(t)$ is the floor acceleration response history, while $t_2 - t_1$ is the time frame that is considered for the numerical integration. While there is still not a clear consensus regarding the time frame that should be considered for the $RMSa$ calculation, the entire history duration is considered herein (Cai et al., 2020).

Figure 7 summarises the results of the load model effect investigation. It is shown that while accounting for the lateral distance between two consecutive footfalls does not seem to modify the response, time overlap can considerably alter the numerical predictions. With that in mind, and in view of the theoretical superiority it offers, the load model where both lateral distance and time overlap are thoroughly considered is employed in the computations presented hereafter.

The investigated floor, on account of a realistic range of walking frequencies could respond reasonable to the fourth ($f_1/4$=1.97Hz) and the fifth ($f_1/5$=1.57Hz) harmonic of the footfall excitation. Figure 7 shows that a local maximum with respect to both the peak and the RMS vertical acceleration is being reached at a frequency that is slightly higher than 2.0Hz. The latter approximately corresponds to the fundamental frequency of the floor divided by four. It should be noted that local peak acceleration values occur in step frequencies higher than the resonant ones, an effect that could be attributed to the adopted load model. This shift of the local peak is consistent with the results reported before by Cai et al. (2020) on account of the same load model.

Probabilistic modelling

In this study the distribution proposed by Chen et al. (2014) for normal walking frequency is adopted [Figure 3(b)] and the pacing frequency is assumed to be constant for all steps in a single footpath. The latter is deemed to be a reasonable simplification despite the fact that it is highly unlikely for an individual to maintain the exact same pace in each footfall (especially if the walking is not guided by a metronome), with maximum recommended deviations from the reference step frequency values in the order of 0.15Hz (Sedlacek et al., 2006).To avoid unrealistic samples that the (full) normal distribution would produce, the respective truncated distribution is employed using lower and upper bounds equal to 1.5Hz and 2.5Hz, respectively, as shown in Figure 8.

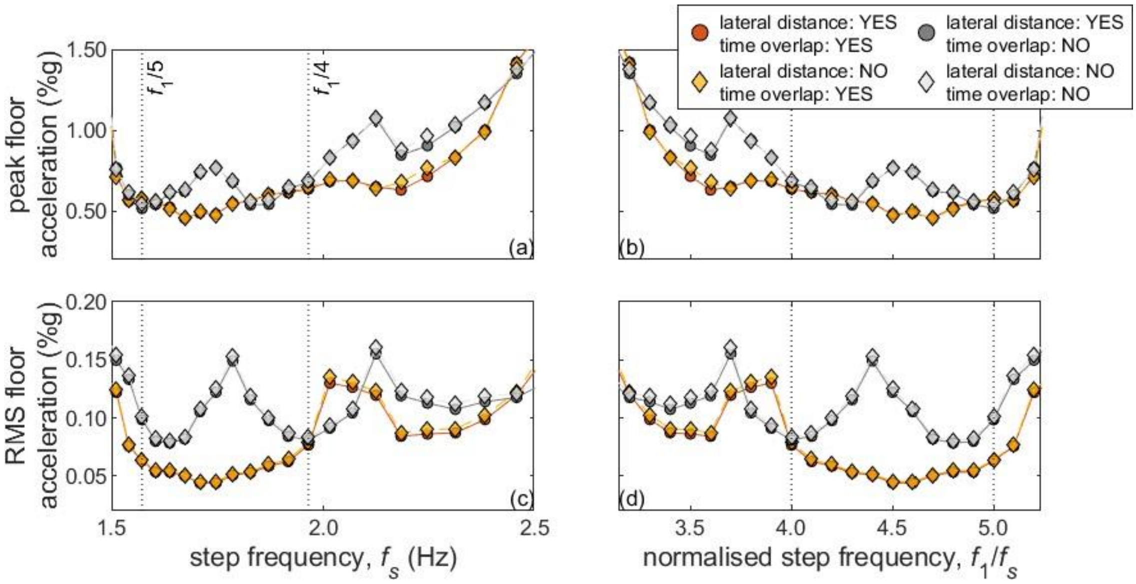

Figure 7: Response spectra of peak (a, b) and root mean square (c, d) acceleration versus step frequency (a, c) and normalised step frequency (b, d), featuring the case study Floor A from Mello et al. (2008)

In this first attempt to probabilistically evaluate the response of the considered floors, human weight is assumed to be constant. Hence, as per the AISC DG11 (AISC, 2016), a weight equal to 700N is considered. Yet, it should be noted that the probabilistic treatment of the walking frequency results in the walking force and the duration of the footfall being also random variables, since they are both functions of the walking frequency in the adopted load model.

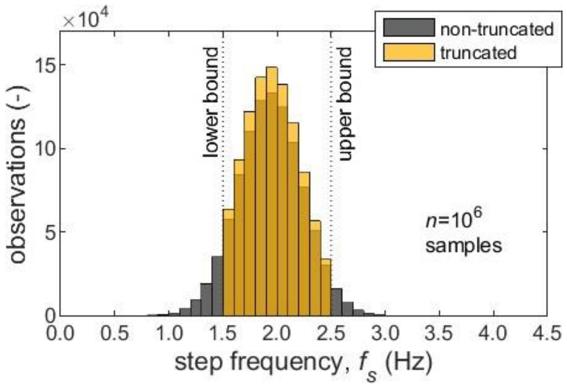

Figure 8: Non-truncated (Chen et al., 2014) versus truncated step frequency distributions

Figure 9 summarises the procedure that has been followed to evaluate the performance of the two case study floors (Table 1). For each floor, a polynomial fit is performed on the peak floor acceleration analysis data [Figure 9(a1, a2)], in view of obtaining an analytical model that allows the generation of a considerable number (i.e., n=10^6, Figure 8) of acceleration responses using the (truncated) step frequency samples shown in Figure 8. Figure 9(b1, b2) show the histograms of the peak floor accelerations that stem from the polynomial fit models, featuring the cases where the AISC (2016) limit is exceeded. The associated cumulative probabilities are also provided in Figure 9(c1, c2). It should be noted that the AISC (2016) limit is considered being deterministic.

Referring to Floor A, the vast majority of peak floor acceleration samples exceed the 0.5%g limit, as shown in Figure 9(b1). This comes as no surprise, since the design of this particular floor is borderline acceptable per AISC (2016), as reported before by Mello et al. (2008). Still, even for the case of Floor B, which is, at least in theory, a more robust system (Chen et al., 2018), more than 50% of the samples exceed the AISC (2016) limit.

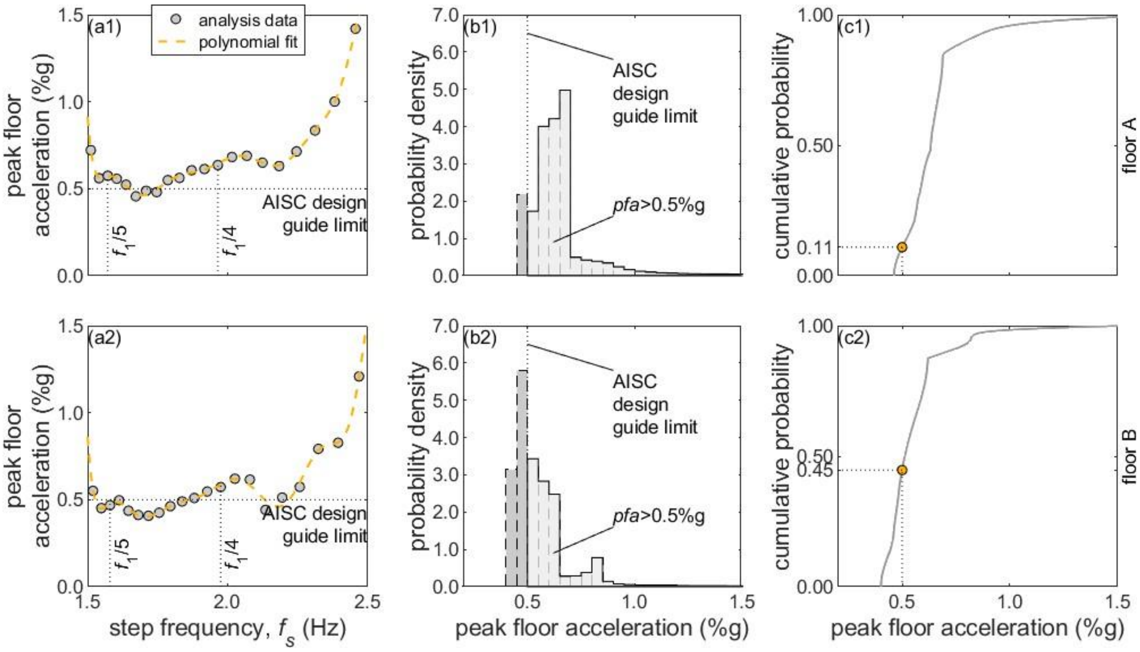

Figure 9: Probabilistic assessment of discomfort, using the floor case studies of Table 1; (1st column) peak floor acceleration data and polynomial fit; (2nd column) probability density, featuring the samples that exceed the 0.5%g limit; (3rd column) cumulative probability

Conclusions

A methodology has been presented to probabilistically evaluate the serviceability performance of single-unit steel-concrete composite floors designed to AISC under human-induced walking excitations. Using a structural model that relies on the grillage technique to model the steelconcrete composite slab, peak floor accelerations are extracted and compared to the 0.5%g AISC (2016) proposed limit for office floors. Employing polynomial fits to approximate the acceleration response versus the walking step frequency that are generated for each case study floor, allows the simulation of a large number of acceleration demand samples considering the actual randomness of the step frequency. This process eventually enables the robust definition of the peak floor acceleration demand distribution. Accounting for the variability in step frequency excitation, reveals a considerable number of floor acceleration exceedances vis-à-vis the 0.5%g limit. The presented methodology could be expanded to accommodate other sources of uncertainty that are deemed to be important (e.g., damping, human weight, modelling etc.), to eventually provide the full picture of what one should expect regarding the level of discomfort that is likely to be encountered in a floor that conforms to contemporary design guidelines.

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