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ANALYSIS OF THE SEISMIC RESPONSE OF EARTH DAMS

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Abstract: *The seismic analysis of dams constitutes a particularly difficult engineering task. Complicated dynamic phenomena such as fluid-structure and soil-structure interaction take place in this problem affecting the response of the dams. Such phenomena can be proved destructive for the dam body, and consequently a catastrophic threat for the downstream communities. This study attempts to analyse the dynamic behaviour of earth dam including the dam-foundation-reservoir interaction effects. In view of this development, Finite Element Method (FEM) models are developed, with the focus to be on their nonlinear response. A parametric study is carried out in regards to the geometrical parameters of a heterogeneous earth dam formed by sandy soil shells and impervious material core. Finally, the numerical models are subjected to 4 well-known ground motions and the resultant response is presented herein. A comparative analysis between a linear and a nonlinear model points out the similarities of their response for moderate intensity earthquakes. Nevertheless, this cannot be stated for higher intensities, as stiffness degradation effects take place.*

Introduction

Failures of dam structures are proved to be a catastrophic threat for the downstream populations and facilities, as flooding can be induced from their failure. In the recent past, such phenomena have occurred due to earthquakes, the last one in Fujinuma, Japan. Twenty minutes after the Tohoku ground motion, in 2011, significant settlements occurred at the crest of the dam resulting in flooding downstream. Thus, the result of the drop of the freeboard was 8 casualties and led to the loss of many properties. Information regarding this event can be found in the literature [Abbas et al. (2011); Pradel et al. (2013)].

A well-known case-study by Seed (1981) concerning the failure of San Fernando earth dam, highlights all the important factors in regards to the earthquake fragility of earthen dams. The homonymous ground shaking in 1971 of a 6.5 magnitude, triggered liquefaction inducing the upstream sand shell of the Lower San Fernando dam to slide. In the same study, Seed (1981) reported that the earthquake performance of earthen dams depends principally on their construction material, the saturation level and the inclination of both slopes. Thus, dams which are made of rockfill materials under drained conditions or clayey materials over firm foundations can resist stronger ground motions. Hence, Seed concluded that research investigations should emphasise on the earthen dams built with cohesionless materials.

Numerous studies have been published regarding the seismic response of earthen dams with analytical and numerical methods concerning their shear dynamic behaviour [Ambraseys and Sarma (1967); Dakoulas and Gazetas (1985); Gazetas and Dakoulas (1992)] or with numerical methods [Elgamal (1992); Blázquez and López-Querol (2007); Pelecanos et al. (2018), etc.]. An extraordinary role on these modelling approaches was played by Westergaard (1933), who introduced the hydrodynamic forces of the reservoir system to the dam with equivalent masses in the analysis. The improvement in the accuracy of this hydrodynamic analysis came from the use of Finite Element Method (FEM) and more specifically with the finite fluid elements. Therefore, currently the researchers are of the ability to carry out the fluid-structure interaction effect using the Eulerian approach [Chopra (1968); Fenves and Chopra (1984); Hall and Chopra (1982)] or with the Lagrangian approach [Wilson and Khalvati (1983);Bayraktar et al. (1994);Bilici et al. (2009)]. The analysis of the unbounded systems of the reservoir or of the foundation can be carried out simulating them as finite with artificial absorbing boundary conditions [Lysmer and Kuhlemeyer (1969); Kellezi (2000); Kontoe et al. (2009)] or with infinite elements [Ungless (1973); Bettess (1977)]. In this way, the dynamic stiffness matrices of the soil and fluid media tend to

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provide continuity to the system describing the behaviour of the infinite domain, in contrast with the elementary boundary conditions which completely reflect the propagating waves.

Methodology

The aim of this paper is to bring into light the earthquake performance of earthfill dams which consist of sandy shells and impervious cores. In such earth structures, significant material nonlinearities take place, inducing failures or high strains which can result in flooding. In order to capture such effects, a nonlinear transient analysis is carried out, with the ultimate objective to be on a geometrical parametric analysis of an earth dam with given material properties.

Numerical modelling

For the evaluation of the seismic response of the earth dams, FEM models are developed considering the dam-foundation-reservoir interaction effect within ANSYS APDL. In view of this development, 2D simulations have been carried out, using the plane strain theory. The numerical model consists of two domains: the dam-foundation with plane elements (Plane 182) and the reservoir with fluid elements (Fluid 79).

Emphasis is given to the simulation of their interfaces which is of great importance for the resultant dynamic behaviour, and for this purpose coupling between both domains has been employed. Thus, the dynamic interaction of the dam-foundation system with the reservoir is expressed in terms of nodal displacements capturing the sloshing effects of the latter. In order to achieve this in detail, the nodes of the two systems at the interface GH (Fig.1) are rotated according to the angle of the slope. In this way, the reservoir is free to oscillate in parallel to the upstream slope. On the other side, the nodes of the reservoir at the interface BG are constrained in the vertical direction with the corresponding ones of the foundation, and thus they are free to oscillate in the horizontal one.

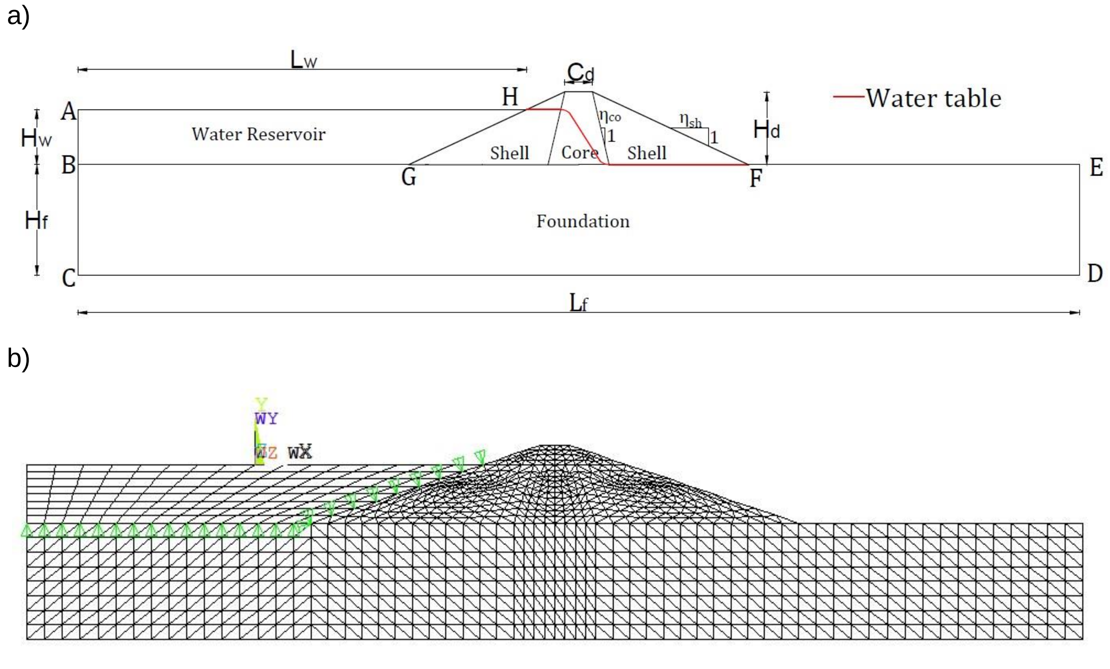

Figure 1 Earth dam: a) Geometry b) FEM mesh

Continuity to the unbounded domains of the foundation and the reservoir is guaranteed by introducing artificial boundary conditions to the finite system. In order to avoid permanent displacements with the commonly used absorbing boundary conditions [Lysmer and Kuhlemeyer (1969)], the cone boundaries are implemented. Nevertheless, the dynamic stiffness of this type of element are function of the distance from the source of excitation, which is the fault in case of earthquakes, which brings into light their drawback. The wave propagation through the reservoir domain reflect to the dam, and thus it can be assumed that this is the source of excitation [Kontoe et al. (2009); Pelecanos et al. (2013)]. In this study, the same assumption is considered also for the foundation medium. Thus, the artificial boundaries are applied perpendicularly to the side abutment of the reservoir (AB) allowing it to oscillate in the vertical direction. On the other hand, the stiffness-dashpots elements are oriented normally and tangentially to the side abutments of the foundation (BC and DE). In the nodes at the bottom of the latter (CD), elementary boundary conditions have been applied. The governing equations for these artificial boundary conditions are presented next:

$$\sigma = \rho \cdot \frac{V_P^2}{2 \cdot r} \cdot u \tag{1}$$

$$\tau = \rho \cdot \frac{V_S^2}{2 \cdot r} \cdot v \tag{2}$$

$$\sigma = \rho \cdot V_P \cdot \dot{u} \tag{3}$$

$$\tau = \rho \cdot V_S \cdot \dot{v} \tag{4}$$

where, σ : normal stress to the plane section, τ : tangential stress to the plane section, ρ : density, r : distance from the source of excitation, V_P : P-wave velocity, V_S : S-wave velocity, u : normal displacement to the plane section, v: tangential displacement to the plane section, $\dot{u}$: normal velocity, $\dot{v}$: tangential velocity

As a consequence of the above, first and foremost, the dimensions of the system should be selected. Concerning the reservoir, the ratio of its length to its depth (L_w/H_d – see Fig.1) is selected to be approximately 8, whereas the ratio of the water level to the height of the dam (H_w/H_d) is assumed to be 0.75. The corresponding ratio of the depth of the foundation to the height of the dam is equal to 1.5 (or twice the height of the reservoir). Finally, the described geometrical parameters are parameterised as a function of the dam height (H_d) and the slope inclination (η_{sh}). This parametric analysis is carried out investigating four different plane sections of dams, with heights of 40 m and 80 m. Regarding the slope inclination, a reasonable value of 1:3 (vertical: horizontal) and a theoretical value of 1:2 (unfavourable conditions) are selected. The slope of the core is kept constant though the parametric analysis and equal to 1:0.2.

Constitutive model

The focus is on methods accounting for material and geometrical nonlinearities. The complex dynamic behaviour of the soil is considered within a simplified constitutive model. Richart et al. (1970) summarised the parameters which affect the shear modulus of soils. Hardin (1978) proposed an equation regarding the effect of the mean effective stresses in the shear modulus in small strain (Gmax). In this study, this equation is taken into account for the estimation of the shear modulus evaluating the mean effective stress of each element separately. Consequently, the water tab`1`le has significant role to this analysis as it affects the initial effective pressures within the dam. Thus, the upstream slope and the core are assumed saturated, but not the downstream slope, Figure 1(a). The soil model parameters of the sand shells and the impervious core are defined after Blázquez and López-Querol (2007), as they can be found in Table 1. Finally, it is worth noting that the initial conditions due to gravity loads are taken into consideration in the dynamic analysis.

$$G_{max} = \frac{B_g \cdot P_a}{0.3 + 0.7 \cdot e^2} \cdot \left(\frac{p'}{P_a} \right)^{0.5} \tag{5}$$

where, G_{max} : shear modulus in small strains, B_g : soil model parameter, e : void ratio p' : mean effective stress, P_a : atmospheric pressure

Dynamic shear stresses are of the ability to trigger liquefaction, as the evolution of the volumetric strain in the elements results in increases of pore water pressure. When that happens, the effective stresses of the solid skeleton tend to zero. Hence, according to the aforementioned dependency of the shear modulus on the effective stresses, the stiffness of the soil elements follows the same trend. In simple words, as the pore water pressure increases, the connection forces between the soil particles drop down which leads to reduced soil stiffness. In order to take into consideration this dynamic behaviour of the soil material, degradation curves are employed in the numerical model as a simplified approach. For the sandy shells and the impervious core, the degradation curves of stiffness and damping by Seed et al. (1986) and Vucetic and Dobry

(1991) are considered respectively. Such effects are assumed negligible in the alluvium foundation.

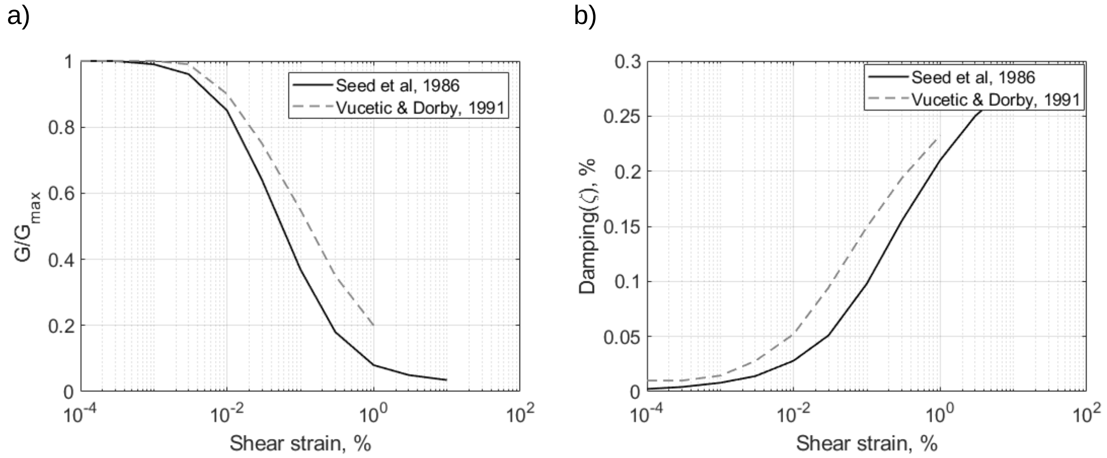

Figure 2 Degradation curves: a) Stiffness degradation b) Damping

The stiffness degradation of each element is a function of its average maximum shear strain. Under large strain conditions, the minimum shear modulus (Gmin) is restrained to 10% of the initial value which is given to each element (Gmax) in order to avoid numerical instabilities in the models. The ratio of the critical damping in each step determines the Rayleigh damping coefficients, which is defined by means of elastic behaviour considerations, and they are estimated by equation 6. The selection of the j-th eigen-frequency is carried out in such manner that the cumulative effective to total mass fraction exceeds 65%, that is to say, the fifth eigen-frequency. At this point, it should be noted the contribution of Woodward and Griffiths (1996), who discussed the limitations of Rayleigh damping in earth structures.

$$\begin{Bmatrix}\alpha\\ \beta\end{Bmatrix} = \frac{2\cdot\zeta}{\omega_i+\omega_j}\begin{Bmatrix}\omega_i\cdot\omega_j\\ 1\end{Bmatrix} \qquad (6)$$

where, α: mass matrix coefficient, β: stiffness matrix coefficient, ω_i: i-th natural eigen-frequency

The damping of the fluid elements is assumed 1% of the critical in order to avoid unrealistic reductions in the hydrodynamic forces Pelecanos et al. (2018). The validation of the dam-reservoir interaction effect is performed using simplified formulas describing the natural-frequency of the system R. D. Blevins (1979).

The simple Mohr-Coulomb yield surface is adopted to describe the stress-strain response of the soil. Woodward and Griffiths (1996) in their study illustrated the benefits and the limitations of such constitutive model. Consequently, the shells of the earth dam are sandy, with 5 kPa cohesion (c), 35º friction angle (φ) and density (γ) of 2.2 ton/m^3. Regarding the impervious soil of the core, it is assumed as clayey soil with 30 kPa cohesion, 5º friction angle and density of 2 ton/m^3. The dam is assumed to be founded on a homogeneous alluvium layer of 200 MPa shear modulus. Information regarding the applied soil parameters can be found in the following references [Elgamal (1992); Pelecanos et al. (2018)].

	Sandy Shell	**Impervious core**	**Foundation**
c (kPa)	5	30	5
φ (º)	35	5	35
γ (ton/m^3)	2.2	2	2.1
B_g	220	170	-
e	0.84	0.75	-

Table 1 Material model parameters

Validation of the numerical model consisted of simplified constitutive law

For the validity of this work, the numerical models are compared with field measurements of recorded accelerations at the Long Valley Dam due to Mammoth Lake earthquake, 27 May 1980. Information regarding the geometry, the material properties of the mentioned dam and the earthquakes can be found in previous studies Lai and Seed (1985) and Woodward and Griffiths (1996).

The comparative results of this analysis present in Figure 3 are referred to the horizontal and vertical accelerations at the crest of the dam and specifically in the central plane section. As it can be observed, the numerical analysis of this study predicts with adequate accuracy the actual response of the dam in the horizontal direction. Nevertheless, the resultant response in the vertical direction presents deviations, as it also observed in the study of Woodward and Griffiths (1996). The authors of the same study pointed out that such constitutive model may be appropriate to describe the response of dams without significant damages in moderate strong earthquakes. In any other case more sophisticated soil models are required.

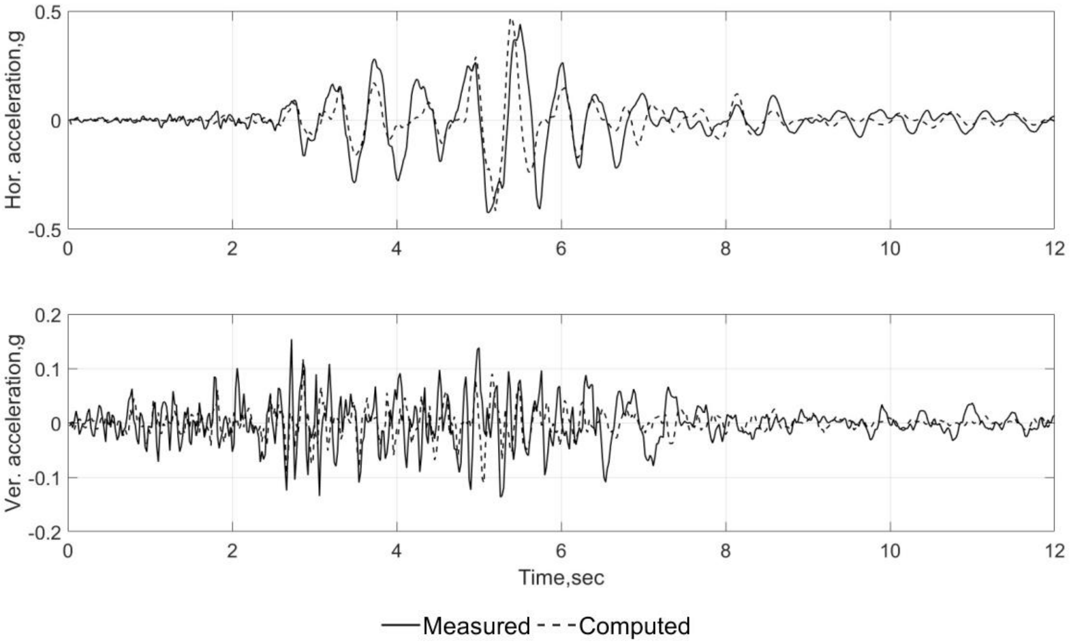

Figure 3 Comparison between measured and computed of horizontal and vertical accelerations at the crest of Long Valley dam

Dynamic behaviour of earth dams

The dynamic behaviour of earth dams is investigated herein. Initially, modal analyses of the dam body and the dam-foundation system are performed excluding the interaction with the reservoir. Table 2 illustrates the Eigen-periods of the examined systems, whereas in Figure 4 a representative Eigen-shape of the examined dams is presented. As it can be observed from Table 2, the height of the heterogeneous dam-foundation system has an important role to their Eigen-periods. This is due to the different geometrical parameters, but also to the material properties which are function of the mean effective stresses. On the other hand, deviations in the Eigen-periods of the dams due to the slope angle are of minor importance, and therefore they are not included in this analysis.

	Height of dam body 40m	**Height of dam body 80m**
Dam body	0.78 sec	1.36 sec
Dam-foundation system	1.29 sec	2.52 sec

Table 2 First eigen-periods of examined dams

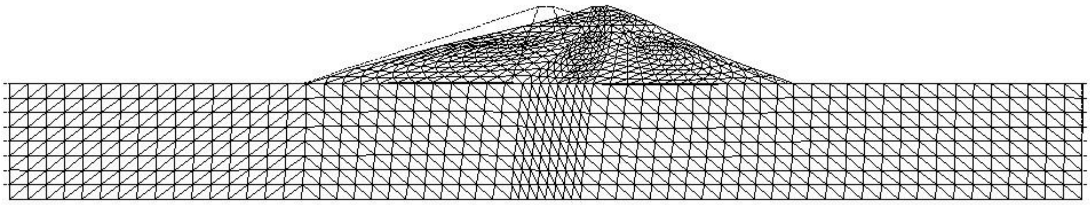

Figure 4 First eigen-mode of the dam-foundation system

Significant influence to these properties of an earth dam can be noticed under large strain conditions. This is mainly observed due to the evolution of nonlinearities in the solid skeleton. Consequently, a comparative analysis of the transient response is performed between a linear model and the one considering the aforementioned constitutive law. The described numerical models of earth dams are subjected to 4 well-known strong ground shakings, as they are plotted in Figure 5. Emphasis is given on the response at the crest of the dam in this analysis. For this purpose, the response spectra (5% of critical damping) of the total horizontal accelerations of this location are plotted in Figure 6.

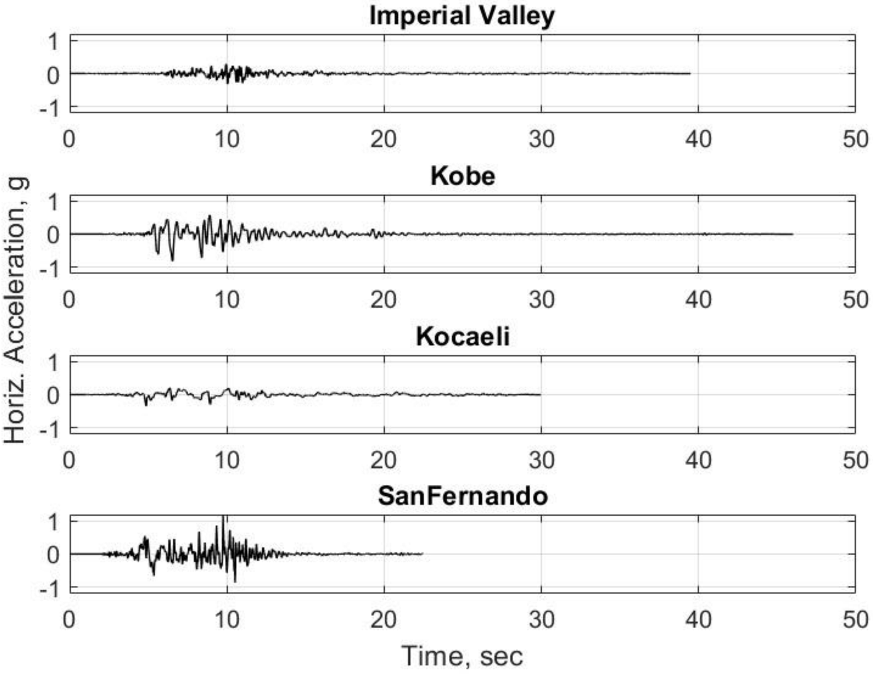

Figure 5 Ground motions

The response of a structural system depends on numerous factors during a dynamic excitation. An important role to this has the dynamic stiffness matrix of the structural system which are incrementally evaluated due to the material or geometrical nonlinearities (large strain analysis). Besides that, the seismic excitations not only vary in amplitude of accelerations in the time domain for a finite duration, but also and more importantly in their frequency content. Hence, different amounts of effective mass can be activated for different earthquakes, and consequently proportional inertial forces are applied to the system. In simple systems, the spectra accelerations of the first Eigen-mode ($S_a(T_1)$) provide significant information regarding this. Nevertheless, in complicated systems, such as in this case of the dam-reservoir interaction, this kind of analysis may not be so accurate. In addition, the accelerations of a nonlinear system may be different from the corresponding ones of a linear system, as its Eigen-properties are greatly affected.

Hence, differences can be observed at their response spectra, which are mainly depended on the intensity of the earthquake, but also to the strain level of the dam body. This is line with the results presented in Figure 6. Looking in detail the response spectra of the dams, main conclusion can be derived. Firstly, the dynamic amplification occurs at the natural period of the dam body and not in the corresponding one of the dam-foundation system. This implies that the foundation does not affect significantly the response of the dam body. Only in the case of the San

Fernando earthquake the spectra accelerations are high enough to enable modes which are involve the foundation of the system. As it was expected, the dynamic amplification of the nonlinear system takes place at higher periods than the linear one having smaller amplitude of spectra accelerations.

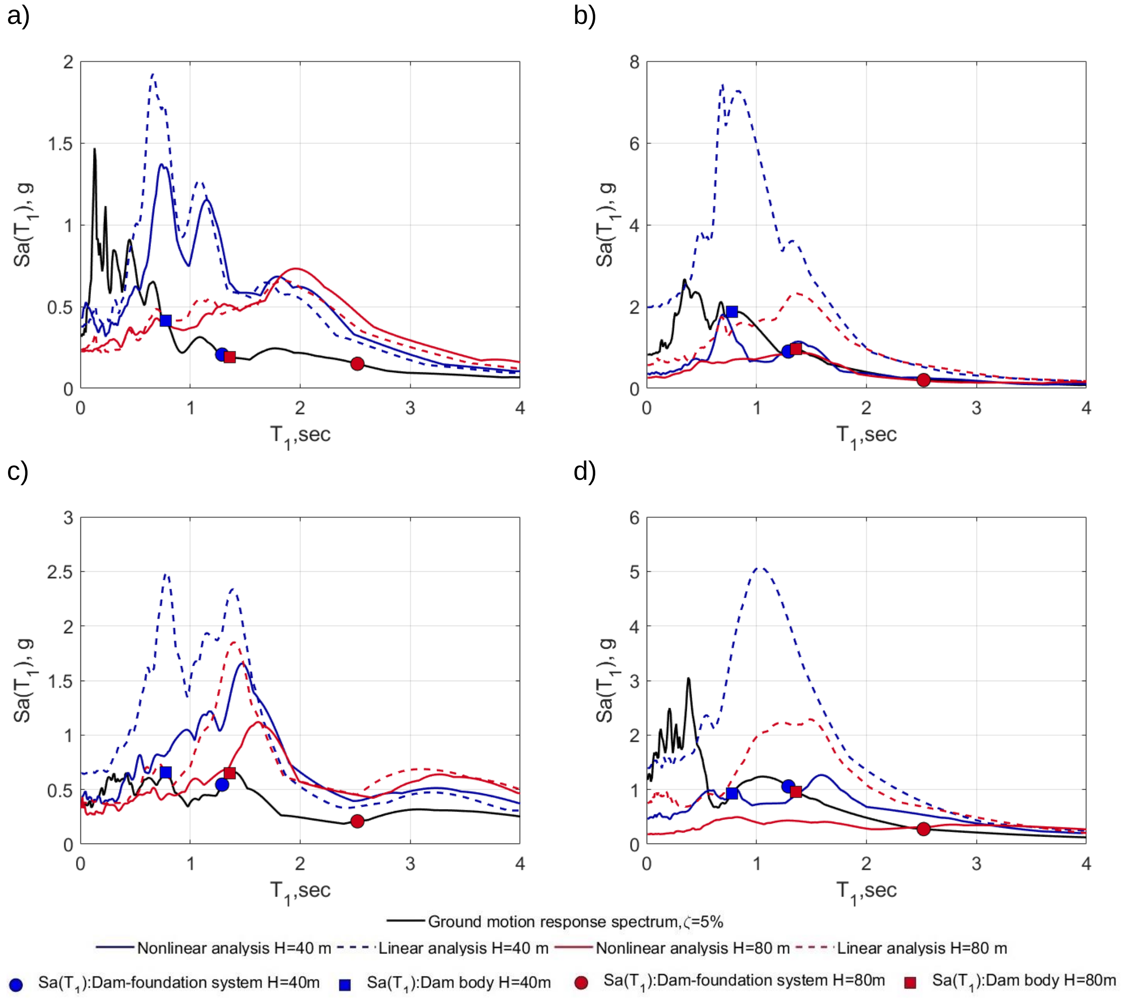

Figure 6 Response spectra (ζ=5%) at the crest of the dam from nonlinear and linear models subjected to ground motions: a) Imperial Valley b) Kobe c) Kocaeli d) San Fernando

It is recalled that this study focuses mainly on the nonlinear response of the of the earth dams. For this purpose, vertical profiles regarding the amplitude of horizontal displacements and total horizontal accelerations of the dam body are presented in Figure 7. It should be noted that the slope of the dams has a minor effect on the total acceleration of the dam body. In detail, the dams with steeper slopes experience smaller accelerations. This implies that the accumulation of plastic strains occurs faster in their solid skeleton, which leads them to oscillate under lower frequencies. Therefore, their nonlinear transient analysis predicts greater displacements that can reach 3 times greater values than a dam with reasonable slope (i.e. 1:3). Another important factor of this analysis is the dam-foundation interaction. The amplitude of total accelerations at the base of both dams with different height are similar for the corresponding seismic excitations. Nevertheless, high intensity earthquakes induce nonlinearities to the foundation of the taller dam, and thus the dam body experiences smaller accelerations at its base.

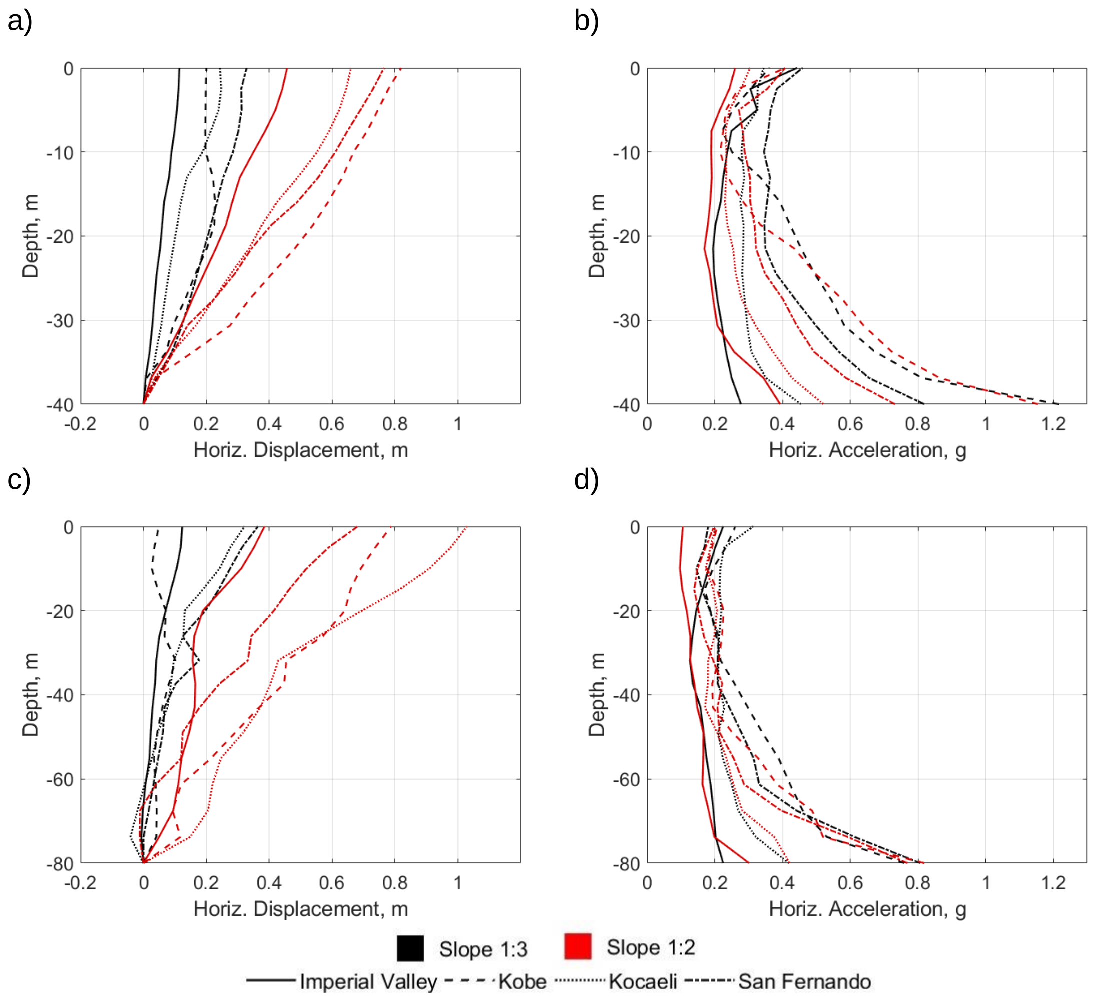

Figure 7 Vertical profile of the amplitude of horizontal displacements and total accelerations: a,b) Height of the dam 40 m c,d) Height of the dam 120 m

Figure 8 illustrates the evolution of settlements at the crest of the dam bodies induced by the seismic excitations presented in Figure 5. There are numerous factors which affect the level of settlements of an earthen dam during an earthquake, from its material properties to its geometrical definition. In regards to the latter, emphasis should be given to their slope as it might affect significantly the level of settlements. In contrast with that, the height of the dam has a smaller contribution to this effect. Nevertheless, dams with the same slope but with different height experience slightly greater settlements, although they display smaller spectral accelerations. Consequently, seismic design considerations, concerning the geometrical parameters of dams which affect their stability, should mainly emphasise their slope inclination and later their height.

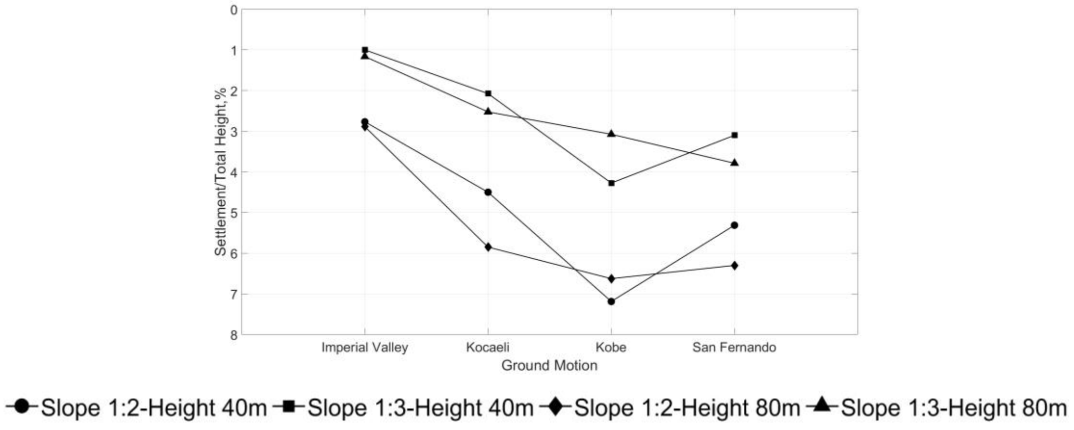

Figure 8 Normalised settlements for the examined ground motions

The residual deformations induced by San Fernando ground motion for all the examined dam bodies are presented in Figure 9. Firstly, important concentration of settlements is estimated for the core of the dam body. Nevertheless, its residual horizontal deformations remain in low level. Hence, it can be concluded that the impervious core provides significant lateral resistance to the dam body. On the contrary, slide movements of the sandy shells are observed, especially in the downstream slope. As it was expected, the amplitude of residual deformations is amplified with the increase of the slope angle.

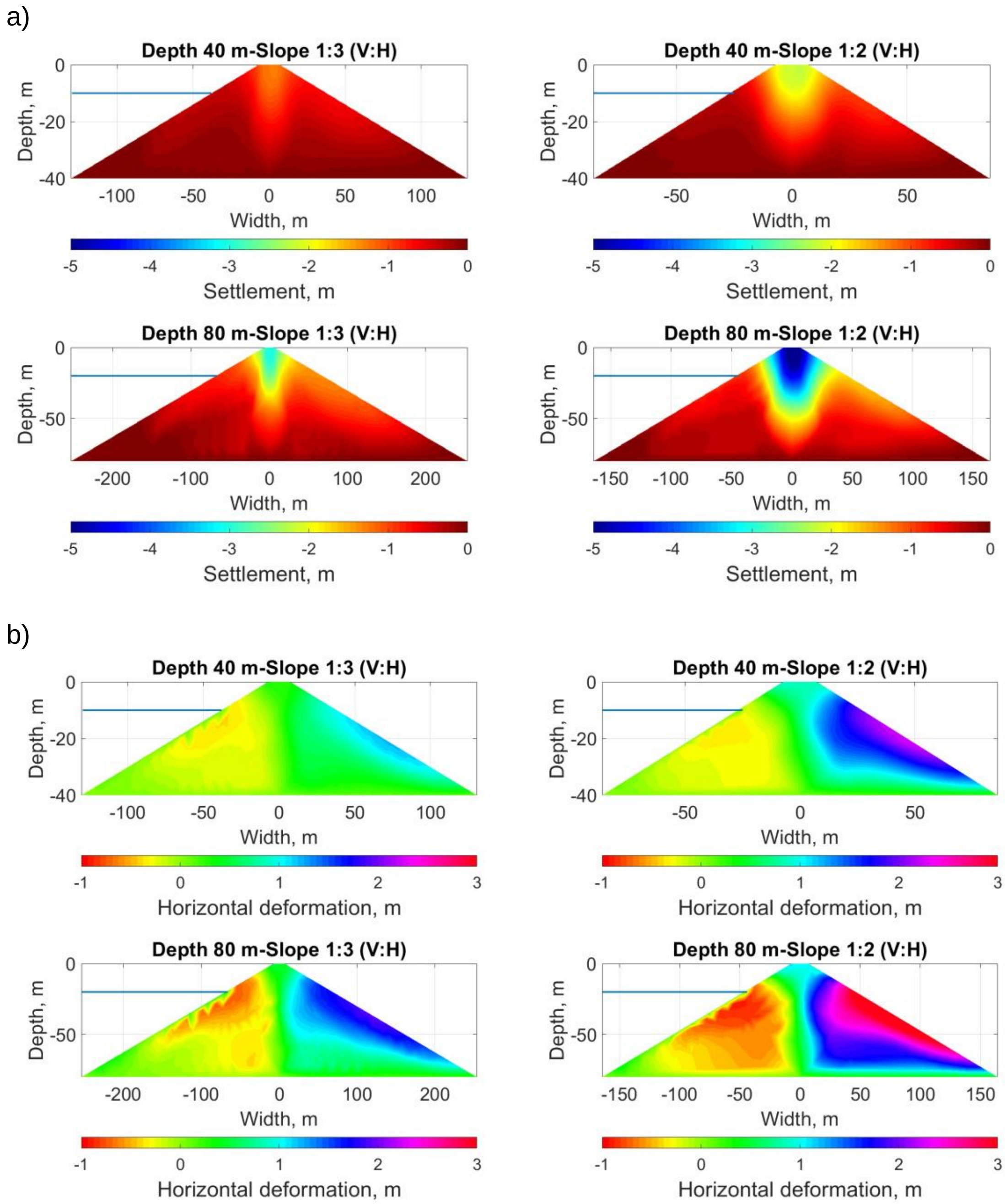

Figure 9 a) Settlements b) Residual horizontal deformation at the end of San Fernando earthquake

Conclusions

The analysis of the seismic response of earthen dam bodies is performed herein. For this purpose, FEM models are developed including the dam-foundation-reservoir interaction effect. Initially, the Eigen-properties of the dam-foundation system are investigated and a comparison of linear and nonlinear analysis is carried out focusing on the dynamic behaviour at the crest of the dam. Subsequently, emphasis on the nonlinear response of the earthen dams is given and important conclusions are derived.

The Eigen-analysis of the dam-foundation system provides significant information in regards to their dynamic behaviour. In addition, this analysis can be proved useful tool for a comparative investigation between nonlinear and linear models. Initially, it should be stated that the Eigen-properties of the dam-body are mainly depended on the height and the stress-dependent elastic material properties. It is worth noting that the dynamic amplification factor of the examined dams is estimated around the natural frequency of the dam body, but not of the whole system, for 3 out of 4 of the examined excitations. Therefore, the foundation has minor effect in the response of the dam body, especially for moderate intensity earthquakes. In addition, dams made with reasonable slopes subjected from low to moderate intensity earthquakes can be analysed in terms of linear analysis. This conclusion is derived from the comparison of the total horizontal accelerations at the crest of the dam for both linear and nonlinear models.

Having a thorough investigation in the nonlinear response of earth dams, numerous conclusions can be derived. First and foremost, it should be noted the importance of the dam-foundation interaction effect. Dams constructed of different geometrical properties experience similar accelerations at their base for small to moderate intensity earthquakes. Nonetheless, deviations in the amplitude of accelerations due to strong ground shakings may appear as nonlinearities might take place in the foundation. Therefore, emphasis should be given on this phenomenon, especially when the dam is founded on materials that are highly susceptible to degradation effects.

Focusing on the response of the dam bodies, it can be stated that both material and geometrical properties have an important role. The impervious core contributes to provide horizontal resistance to the dam body, although local settlements are located at their top. In regards to the slope angle of the sandy shells, on the one hand, it has negligible influence in the total acceleration of the dam body. On the other hand, the residual deformations are amplified as the dam has steeper slopes.

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